2014. Detailed program **REALIZED** of the course

DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS[1], [2]

Week 1.

Lecture (February12, 2014)

1. Systems of ordinary differential equations (ODE). Integral curves, phase portraits (in case of autonomous systems. Initial value problem, Lipschitz condition. Theorem of existence and uniqueness (only formulation). Gronwall lemma (I. Bihari). Proof of uniqueness.
2. First order separable equations. Phase portraits if one dimensional autonomous systems. Examples.
3. Short test (to have some information about the knowledge of the students –collected, but without grades and detailed correction).

Tutorial (practical lecture, February 10, 2014)

1. Integrable and separable ODEs.
2. Algebraic methods: Taylor-series method and the method of undetermined coefficients.
3. Explicit Euler method.

Week 2.

Lecture (February 19, 2014

1. Systems of linear ordinary differential equations (ODE). General case. The maximal interval of solutions coincides with the domain of definition with respect to t of the right hand side (proof by Gronwall lemma). Homogeneous systems (H). Fundamental matrix, system. Wronski determinant. Non-homogeneous systems (IH). Method of variation of constants. Theorem of Structure (General solution of (IH) = General solution of (H) + Particular solution of (IH)).
2. First order linear ODE. Formula for general solution. Example.

Tutorial (practical lecture, February 17, 2014)

1. Linear ODEs (solved by the multiplication trick).
2. Explicit Euler method applied to the test equation. Implicit Euler method.
3. Handling the implicitness: introduction to the functional iteration and Newton's method.

Week 3.

Lecture (February 26, 2014)

1. Linear systems with constant coefficients. General solution (by 2 methods: (i) exponential matrix function, eigenvalues, eigenvectors). Phase portraits in 2 dimensional case. Example.
2. Second order ordinary differential equations. Short summary of the general theory. Reducible equations.
3. Linear second order equations with constant coefficients. Examples.

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[1] Code of the subject: BMETE90MX46, Contact hours: 4 lectures + 2 tutorials + 0 lab / week, Credit: 8, Evaluation: exam, Prerequisites: BSc diploma, Semester: 2nd (in the academic year), 1st (in the given MSc program)

[2] The lectures will speak about the related results of late Hungarian mathematicians (among others former profs of BME) as well. Their names are written in blue.

Tutorial (practical lecture, February 24, 2014)

1. Linear constant coefficient ODEs (when the particular solution has a special form).
2. Basic results on functional iteration and on Newton's method.
3. Definition of A-stability. Improved Euler method, trapezoidal method, RK4.

Week 4.

Lecture (March 5, 2014, 2014)

1. 2-dimensional autonomous systems. Linearization . Phase space analysis near equilibrium points (linearization, Poincaré theory).
2. Examples (population dynamics, predator-prey equation, M. Farkas).
3. Laplace transformation. Definition. Solving linear equations, systems by Laplace transformation.
4. Pendulum, linearization, harmonic oscillator (solving by different methods).

Tutorial (practical lecture, March 3, 2014)

1. First order linear constant coefficient 2D systems (solution, phase portrait).
2. Explicit Euler, implicit Euler and trapezoidal method applied on a center type 2D system.

Week 5.

Lecture (March 12, 2014)

Lyapunov stability, definitions.
Consistency, 0-stability and convergence of numerical methods. (Consistency + 0-stabiliy = convergence.)
Convergence of one-step methods.
Step size control.

Tutorial (practical lecture, March 10, 2014)

Group 1. (Topics asked by the students.)
Planar linear autonomous systems. General solution in case of complex roots by 3 methods: reduction to 2^{nd} order equation, Laplace transform, matrix).
Euler equations. Reduction to linear equations with constant coefficients.
Group 2. (Topics asked partly by the students.)
Substitutions in ODEs. First order equations t=y/x. Euler equation x=exp(t).
Planar linear autonomous systems. General solution in case of double roots by 3 methods: reduction to 2^{nd} order equation, Laplace transform, matrix).

Week 6.

Lecture (March 19, 2014)

1. Application of the Ljapunov stability and instability theorems.
2. Periodic orbits. The idea of the Poincaré-Bendixson theorem (weak form).
3. TEST 1.

Tutorial (practical lecture, March 17, 2014)

2D nonlinear autonomous systems. Phase portrait, linearization.

Week 7.

Lecture (March 26, 2014)
1. Remarks, solution of some problems of Test 1.
2. Lyapunov stability by the first approximation (linearization). Proof incase of asymptotic stability (based on Gronwall lemma).Routh-Hurwitz criterion.
3. Solving a numerical example.

Tutorial (practical lecture, March 24, 2014)
Bifurcation problems.

Week 8.
Lecture (April 2, 2014)
1. Lyapunov stability by the first approximation. Example (model of the steam engine).
2. Differentiable dependence on initial conditions, parameters. Linearization. Variational system. Example (pendulum).
3. Definition of topological space.

Tutorial (practical lecture, March 31, 2014)

Week 9.
Lecture (April 9, 2014)
1. Differentiable dependence on initial conditions. Example (space ship).
2. Nonlinear problems. 1-codimensional bifurcations (saddle-node).
3. Definition of metric spaces. Banach fixed point theorem (with proof).

Tutorial (practical lecture, April 7, 2014)

Week 10.
Lecture (April 16, 2014)
1. Nonlinear problems. 1-codimensional bifurcations (Andronov-Hopf).
2. Definition of Banach, Hilbert spaces. Fourier series.

Tutorial (practical lecture, April 14, 2014)

Week 11.
Lecture (April 23, 2014)
1. Fourier series. Convergence, expansion.
2. Heat transfer equation. Fourier method.

Tutorial (practical lecture, April 21, 2014)