Mathematics G3 for Mechanical Engineers (BMETE93BG03)

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Prerequisites: Mathematics G1, G2.

Program:

Classification of differential equations. Separable ordinary differential equations, linear equations with constant and variable coefficients, systems of linear differential equations with constant coefficients. Some applications of ODEs. Scalar and vector fields. Line and surface integrals. Divergence and curl, theorems of Gauss and Stokes, Green formulae. Conservative vector fields, potentials. Some applications of vector analysis. (4 hours/4 credits)

Literature:

Thomas' Calculus by Thomas, G.B. et al. Addison-Wesley, Several editions (ISBN0321185587)

Detailed program (Fall 2024)

Week 1.

Lecture 1. (September 9, Tuesday).

Ordinary differential equations are mathematical models for time dependent finite dimensional differentiable deterministic processes.

First order ordinary differential equations, ODEs, $y' = \frac{dy}{dx} = f(x,y)$. Definition of the solution, integral curve, trajectory. Initial value (Cauchy) problem: $y' = f(x,y).y(x_0) = y_0$. Theorem of Existence & Uniqueness for the initial value problem (without proof) with conditions $f, \frac{\partial f}{\partial y} \in C^0(D)$.

Examples (guess the solutions). General solution. . Initial value problems (i) $y' = y, y(0) = 1, y = e^x$. (ii) Maximal interval is "smaller".$y' = 1 + y^2, y(0) = 0, y = \tan x$. Checking. The function: $y = x + \frac{x^3}{3}$ does not satisfy the equation. (iii) $y' = f(x), y = \int f(x)dx + c$.
(iv) There is no uniqueness $y' = 3y^{\frac{2}{3}}, y(0) = 0$.

Special cases. Separable equations. $\frac{dy}{dx} = g(x)H(y)$. (i) $H(y) = 0$, constant solutions. (ii) $\int \frac{dy}{H(y)} = \int g(x)dx + c$ (with proof in case of initial value problem).

Examples. $y' = y, y(0) = 1, y = e^x$. $y' = 1 + y^2, y(0) = 0, y = \tan x$. (See above, solve as separable equations).

Lecture 2. (September 10, 2024, Wednesday).

Separable equation. Another example. Solve the initial value problem, sketch the graphs of solutions $y' = 2x\cos^2 y$, $(i)\ y(0) = \frac{\pi}{2}$, $(ii)\ y(0) = 0$. Solutions$(i)\ y = \frac{\pi}{2}$, $(ii)\ y = \arctan x^2$.

Another special case: Linear equation. $y' + P(x)y = Q(x)$. Solution:

$y = ce^{\int -P(x)dx} + e^{\int -P(x)dx} \int e^{\int P(x)dx} Q(x)dx$ (proof by the variation of constants).

Example (solving linear equations – general solution, initial value problem, sketching the integral curve(s):

$xy' = 3y + x^2, \quad y(1) = 0$. Solution $y = x^3 - x^2$. $0 < x$.

Literature: Thomas' Calculus Chapter 9, § 1,2,5.

Homework (optional): Thomas' Calculus. **9.1**.11,17, **9.2**.15.

Remark: $y' + 2y = 3$, $y(0) = 1$ is both separable and linear equation.