Mathematics EP1. Detailed program (2025):

Week 1.

Lecture 1 (September 10, Wednesday).

Theory.

Numbers (natural, integer, rational, real, complex).
Real numbers. Axioms. Commutativity, associativity, distributivity. Proof of identities, by axioms. $(a+b)^2 = a^2 + 2ab + b^2$, by division of a square.

Complex numbers (definition, algebraic-polar=exponential forms, addition, subtraction, multiplication, division, de Moivre formula). Fundamental theorem of algebra. Multiplication by i, rotation in the plane around origin by 90°.

Exercises.

Practical lecture 1 (September 9, Tuesday).
Real, complex numbers. Basic identities, binomial theorem.
Calculations with complex numbers. Consider the complex numbers: $z_1 = 1 + i = \sqrt{2}e^{i\frac{\pi}{4}}$, $z_2 = 2(cos\,90^0 + i\,sin90^0) = 2e^{i\frac{\pi}{2}} = 2i$. Find the value of the following complex numbers: (i) $z_1 + z_2 = 1 + 3i$, (ii) $z_1 \cdot z_2 = -2 + 2i = 2\sqrt{2}e^{i\frac{3\pi}{4}}$, (iii) $\frac{z_2}{z_1} = \sqrt{2}e^{i\frac{\pi}{4}}$, (iv) ${z_1}^2 = 2i$, (v) ${z_1}^4 = -4$.
Solve the equation $z^3 + 1 = (z+1)(z^2 - z + 1) = 0$.

Practical lecture 2. (September 12, Friday).
Real, complex numbers. Basic identities, binomial theorem.

Complex numbers.
Calculations with complex numbers. Consider the complex numbers: $z_1 = 1 - i = \sqrt{2}e^{-i\frac{\pi}{4}}$, $z_2 = 2(\cos 270^0 + i \sin 270^0) = 2e^{-i\frac{\pi}{2}} = -2i$. (i) $z_1 + z_2 = 1 - 3i$, (ii) $z_1 \cdot z_2 = -2 - 2i = 2\sqrt{2}e^{-i\frac{3\pi}{4}}$, (iii) $\frac{z_2}{z_1} = \sqrt{2}e^{-i\frac{\pi}{4}}$, (iv) ${z_1}^2 = -2i$, (v) ${z_1}^4 = -4$.
Solve the equations $z^2 + 2iz - 5 = 0$ $z^3 - 1 = (z+1)(z^2 + z + 1) = 0$.

Further applications of complex numbers.

(i) Solve the equation (lecture) $z^2 - 2iz - 5 = 0$, $z^4 + z^2 - 2 = 0$, $z^3 + z - 2 = 0, z^3 - 2z^2 + z - 2 = 0$.

(ii) 2 vertices of a square are $z_1 = 0$, $z_2 = \pm 5 + 12i$. Find the other vertices, perimeter, area of this square.

Literature.

Thomas' Calculus. Chapters 1. §1, Chapter 12. §1-3. Appendices: F3. Real numbers, F4. Complex numbers.

Homework.

Thomas' Calculus**. F3** (Appendix). 7., 13., 23. **Chapter 12**. 3.1.