Mathematics EP2 (BMETE90AX34)

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Detailed summary of the lectures:

Lecture 1. (February 14, 2024).

Ordinary differential equations are mathematical models for time dependent finite dimensional differentiable deterministic processes.

First order ordinary differential equations, ODEs, $y' = \frac{dy}{dx} = f(x,y)$. Definition of the solution, integral curve, trajectory. Initial value (Cauchy) problem: $y' = f(x,y). y(x_0) = y_0$. Theorem of Existence & Uniqueness for the initial value problem (without proof) with conditions $f, \frac{\partial f}{\partial y} \in C^0(D)$.

Examples. General solution. $y' = f(x)$, $y = \int f(x)dx + c$. Initial value problem $y' = y, y(0) = 1$, $y = e^x$ (guessed solution).

Special cases.

Separable equation. $\frac{dy}{dx} = g(x)H(y)$. (i) $H(y) = 0$, constant solutions. (ii) $\int \frac{dy}{H(y)} = \int g(x)\, dx + c$ (with proof).

Examples. $y' = y$, (i) $y(0) = 1$, (ii) $y(0) = 0$. Solution (i) $y(x) = e^x$, (ii) $y(x) = 0$.

Linear equation. $y' + P(x)y = Q(x)$. Solution: $y = ce^{\int -P(x)dx} + e^{\int -P(x)dx} \int e^{\int P(x)dx} Q(x)dx$.

Example (solving linear equations – general solution, initial value problem, sketching the integral curve(s): (i) $y' = y$, $y(0) = 1, y(0) = 0$. Solutions $y(x) = e^x$, $y(x) = 0$ . . (ii) $xy' = 3y + x^2$, $y(1) = 0$. Solution $y = x^3 - x^2$. $0 < x$.

Literature: Thomas' Calculus Chapter 9, § 1,2.

Homework (optional): Thomas' Calculus. **9.1**.11,17, **9.2**.15, and sample tests in TEAMS and on my homepage Google: Moson Péter or direct address: http://tutor.nok.bme.hu/sandwich/general/moremoson/mo.htm .

Lecture 2. (February 21, 2024).

Orthogonal trajectories. Cartesian, polar coordinates (original family $y = cx$ $\rightarrow$ orthogonal family $x^2 + y^2 = c^2$) and reversely original family $x^2 + y^2 = c^2$ $\rightarrow$ orthogonal family $y = cx$.

Further example, homework: $y = ce^x$, find the orthogonal trajectories, sketch both families of curves.

First order autonomous differential equation $y' = f(y)$. Trajectories do not intersect each other. Analytic solution. Separable equation. (i) $f(y) = 0, y = y_1, \dots$ constant solutions. (ii) $f(y) \neq 0, \int \frac{dy}{f(y)} = \int dx = x + c$. Geometric solution. Phase portrait (line, trajectories).

Literature: Thomas' Calculus Chapter **9, § 4, 5.**

Homework (optional): Thomas' Calculus. **9.4**.3, 7. , **9.5**.17.

Lecture 3. (February 28, 2024).

First order autonomous differential equation $y' = f(y)$. Phase portrait. Typical integral curves with inflection points ($\frac{df(y)}{dy} = 0$) . Example. $y' = y^2 - y - 2$.

ODEs of the second order. $y'' = f(x, y, y')$, $y(x_0) = y_0$, $y'(x_0) = y_0{}'$. Theorem of existence and uniqueness. Examples: gravity / $my'' = mg$, harmonic oscillator / $y'' + y = 0$.

Linear ordinary differential equations of the second order with constant coefficients. Theorem of structure. General solution of the homogeneous equation (3 cases).
Example: Solution of the harmonic oscillator, as linear equation.
Homework (optional). Find the general solution for the following differential equations $y'' - 2y' + 2y = 0$, $y'' + y' = 0$, $y'' - 2y' + y = 0$.

Lecture 4. (March 6, 2024).
Linear ordinary differential equations of the second order with constant coefficients. General solution of the nonhomogeneous equation (if the right-hand side is a polynomial, exponential, trigonometric function).
Example: Solution of the gravity, as linear equation with constant coefficients.
FSolve the initial value problem $y'' + y' = -1$, $y(0) = 1$, $y'(0) = -2$ by 2 methods (linear equation, Newton's method – until the third order terms). The idea of these methods: (i) Linear with constant coefficients. Homogeneous, characteristic equation, nonhomogeneous, resonance. (ii) Newton's method. We are looking for the solution in form $y(x) = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \cdots = 1 - 2x + \frac{x^2}{2} + \cdots$.
Homework (optional). Solve the initial value problem $y'' + 2y' = 3e^x$, $y(0) = 2$, $y'(0) = 1$ by 2 methods (linear equation, Newton's method – until the fourth order terms).

Lecture 5. (March 13, 2024).
Find the general solution $y'' + y = e^{-x} + \sin 2x + \cos x$. Explanation of the resonance.

2 dimensional systems, autonomous systems, linear systems with constant coefficients. Matrices. Eigenvalues, eigenvectors. General solution. General solution, phase portraits (in case of real eigenvalues – saddle, node.

Homework. (Optional)
Find the general solution of the following differential equation $y'' + y' - 2y = 1 + 2\sin x\ \cos x + 2\cosh x - 2x^2$.

Lecture 6. (March 20, 2024). Planned.

Sample Test 1.

General solution, phase portraits (in case of real eigenvalues – saddle, node - complex eigenvalues – focus, center).

Example: $\dot{x} = -2x + ay,\ \dot{y} = x - 2y, \quad a = -1,\ +1,\ +9.$

Equations - systems. $\ddot{x} + x = 0 \quad \Leftrightarrow \quad \dot{x} = y,\ \dot{y} = -x.$

Homework. (Optional)

Find the general solution, sketch the phase portrait of the system of differential equations $\dot{x} = -3x + 2y,\ \dot{y} = ax - 3y, \quad a = -2, 2, 8$. Remark: systems will not be included to Test1.

Lecture 7. (March 27, 2024).

Test 1. Room K. 221. 8:15 a.m.