Mathematics G3 for Mechanical Engineers (BMETE93BG03)

Lecturer: *Dr. Peter Moson* (www.math.bme.hu, +3614632690, +36309329626)

Prerequisites: Mathematics G1, G2.

Program:

Classification of differential equations. Separable ordinary differential equations, linear equations with constant and variable coefficients, systems of linear differential equations with constant coefficients. Some applications of ODEs. Scalar and vector fields. Line and surface integrals. Divergence and curl, theorems of Gauss and Stokes, Green formulae. Conservative vector fields, potentials. Some applications of vector analysis. (4 hours/4 credits)

Literature:

Thomas' Calculus by Thomas, G.B. et al. Addison-Wesley, Several editions (ISBN0321185587)

Detailed program (Fall 2024)

Week 1.

Lecture 1. (September 3, Tuesday).

Ordinary differential equations are mathematical models for time dependent finite dimensional differentiable deterministic processes.

First order ordinary differential equations, ODEs, $y' = \frac{dy}{dx} = f(x,y)$. Definition of the solution, integral curve, trajectory. Initial value (Cauchy) problem: $y' = f(x,y). y(x_0) = y_0$. Theorem of Existence & Uniqueness for the initial value problem (without proof) with conditions $f, \frac{\partial f}{\partial y} \in C^0(D)$.

Examples (guess the solutions). General solution. . Initial value problems $y' = y, y(0) = 1$, $y = e^x$. Maximal interval is "smaller".$y' = 1 + y^2, y(0) = 0, y = \tan x$. Checking the solution: $y' = 1 + x^2, y(0) = 0, y = x + \frac{x^3}{3}$. Remark (in general): $y' = f(x)$, $y = \int f(x)dx + c$.

There is no uniqueness $y' = 3y^{\frac{2}{3}}, y(0) = 0$.

Special cases. Separable equations. $\frac{dy}{dx} = g(x)H(y)$. (i) $H(y) = 0$, constant solutions. (ii)

$\int \frac{dy}{H(y)} = \int g(x)dx + c$ (with proof in case of initial value problem).

Example. .$y' = 1 + y^2, y(0) = 0$, $y = \tan x$ (solve as separable equation). $y' = 2x\cos^2 y$, $(i)\ y(0) = \frac{\pi}{2}, (ii)\ y(0) = 0$. Solutions$(i)\ y = \frac{\pi}{2},\ (ii)\ y = \arctan x^2$.

Lecture 2. (September 4, 2024, Wednesday).

Another special case: Linear equation. $y' + P(x)y = Q(x)$. Solution:

$y = ce^{\int -P(x)dx} + e^{\int -P(x)dx}\int e^{\int P(x)dx}Q(x)dx$ (proof by the variation of constants).

Example (solving linear equations – general solution, initial value problem, sketching the integral curve(s):

$xy' = 3y + x^2, \quad y(1) = 0$. Solution $y = x^3 - x^2$. $0 < x$.

$y'\cos x + y\sin x - 1 = 0, \ (i)\ y(0) = 0, \ (ii)\ y(0) = 1$). Solutions (i) $y = \sin x$, (ii) $y = \cos x + \sin x \ , -\frac{\pi}{2} < x < \frac{\pi}{2}$.

Literature: Thomas' Calculus Chapter 9, § 1,2,5.

Homework (optional): Thomas' Calculus. **9.1**.11,17, **9.2**.15.

Remark: $y' + 2y = 3, y(0) = 1$ is both separable and linear equation.

Week 2.

Lecture 3. (September10, Tuesday).

Orthogonal trajectories. Theory. Examples: Straight lines passing through the origin, concentric circles centered at the origin – solution in both directions. Cartesian, polar coordinates ($y = cx, \ x^2 + y^2 = c^2$).

Further example: orthogonal trajectories $y = \ln(cx)$, $c > 0$.

Approximate solution of ODEs. Newton's, Euler's methods. Example: $y' = y, \ y(0) = 1$. Newton's method leads to the solution $y(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$. Approximate value of e . Newton: $e = \sum_{k=0}^{\infty} \frac{1}{k!} = 1 + 1 + \frac{1}{2} + \ldots$, Euler: $\left(1 + \frac{1}{n}\right)^n \xrightarrow[n\to\infty]{} e$.

Lecture 4. (September 11, Wednesday)

First order autonomous differential equation $y' = f(y)$. Phase portrait (line, trajectories). Sketching the graph of some typical integral curves (with inflection points). Examples. $y' = y$, $y' = y^2 - 3y + 2$. Further example (homework): $y' = \sin y$.

Approximate solution of ODEs. Newton's, Euler's methods. Example: $y' = x^2 + y^2 - 1$, $y(0) = 0$. Newton's method leads to the solution (until the 3rd order tems) $y(x) = -x + \frac{2}{3}x^3$. The same problem investigated by Euler's, isoclines method.

Literature: Thomas' Calculus Chapter **9, § 3, 4, 5**.

Week 3.

Lecture 5. (September 17, Tuesday).

University Day. No lecture.

Lecture 6. (September 18, Wednesday)

ODEs of the second order. $y'' = f(x, y, y')$, $y(x_0) = y_0$, $y'(x_0) = y_0{}'$. Theorem of existence and uniqueness.

Examples (solution by 2 methods – elementary ideas, later linear equations with constant coefficients): $my'' = mg -$ gravity, $y'' + y = 0 -$ harmonic oscillator.

Linear second order equations. Theorem of structure (general solution of non-homogeneous equation = general solution of homogeneous equation + particular solution of non-homogeneous equation.

Linear equations with constant coefficients. General solution of homogeneous equation (3 cases). Particular solution of nonhomogeneous linear equations with constant coefficients.in special case of polynomial, exponential trigonometric right-hand sides.

Solve the initial value problem $y'' + y' = -1,\ y(0) = 1,\ y'(0) = -2$ by 2 methods (linear equation, reducible equation, Newton's method – until the third order terms). The idea of these methods: (i) Linear with constant coefficients. Homogeneous, characteristic equation, nonhomogeneous, resonance. (ii) Substitution $u(x) = y'(x)$. (iii) Newton's method. We are looking for the solution in form $y(x) = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \cdots = 1 - 2x + \frac{x^2}{2} + \cdots$.

Week 4.

Lecture 7. (September 24, Tuesday).

Solve the initial value problem $y'' + y' = -1,\ y(0) = 1,\ y'(0) = -2$ by 2 methods (linear equation, reducible equation, Newton's method – until the third order terms). The idea of these methods: (i) Linear with constant coefficients. Homogeneous, characteristic equation, nonhomogeneous, resonance. (ii) Substitution $u(x) = y'(x)$. (iii) Newton's method. We are looking for the solution in form $y(x) = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \cdots = 1 - 2x + \frac{x^2}{2} + \cdots$.

Linear equations with constant coefficients. Superposition principle. Find the general solution of $y'' + y = xe^{-x} + \sin 2x + \cos x$.
Reducible by substitution t0 1st order equations 2nd order equations:
y=f(x,y'), u(x)=y'(x), $y = f(y, y')$, u(y)=y'(x) .
Homework (optional):
Solve the initial value problem $y'' + 2y' = 3e^x$, $y(0) = 2,\ y'(0) = 1$ by 3 methods (linear equation, reducible equation, Newton's method – until the fourth order terms).

Lecture 8. (September 25, Wednesday).

Systems of ODEs. Theorem of existence & uniqueness.

Homogeneous linear autonomous (with constant coefficients) systems of ODEs. Eigenvalues, eigenvectors, general solution.

2-dimensional homogeneous linear systems with constant coefficients. General solution with matrix method.

Case of real eigenvalues. Typical phase portraits (stable, unstable node, saddle).

Week 5.

Lecture 9. (October 1, Tuesday).

Example $\dot{x} = 4x - 4y$, $\dot{y} = -9x + 4y$, the phase portrait is a saddle (solved at the lecture).
Another example (Homework optional) $\dot{x} = 4x - y$, $\dot{y} = -9x + 4y$, the phase portrait is an unstable node.
Case of complex conjugate eigenvalues. Phase portraits (stable-unstable focus, center).
Example: $\dot{x} = 4x + y$, $\dot{y} = -9x + 4y$, the phase portrait is an unstable focus.

Homework (optional). Solve the systems of ODEs $\dot{x} = -x + ay + 1,\ \dot{y} = x - y - 1,\ a = -4, +4$
Sketch the phase portraits. Remark: the stationary point is not at the origin.
Consider the system of ODEs $\dot{x} = -x + 1$, $\dot{y} = x - y - 1$. Solve the initial value (Cauchy) problem $x(0) = 2,\ y(0) = 0$

Lecture 10. (October 2, Wednesday)

Sample Test 1.

Week 6.

Lecture 11. (October 8, Tuesday, 8:15).
Test1. R. 501.

Lecture 12. (October 9, Wednesday).

Scalar fields (invariant, Cartesian, cylindrical, spherical coordinate form). Derivation (gradient, Nabla operator). $u = u(\bar{r}) = u(x, y, z) = u(r, \varphi, h) = u(R, \theta, \varphi)$. Example: $u(\bar{r}) = \bar{r}^2 = x^2 + y^2 + z^2 = r^2 + h^2 = R^2$ Vector fields $\bar{v}(\bar{r}) = v^1(x,y,z)\bar{\imath} + v^2(x,y,z)\bar{\jmath} + v^3(x,y,z)\bar{k}$. Nabla operator: $\nabla = \bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{k}\frac{\partial}{\partial z}$. Derivation. Gradient (vector field): $\nabla u = \bar{i}\frac{\partial u}{\partial x} + \bar{j}\frac{\partial u}{\partial y} + \bar{k}\frac{\partial u}{\partial z}$), derivative tensor (matrix), divergence (scalar field): $\nabla \cdot \bar{v} = \frac{\partial v^1}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial v^3}{\partial z}$.

Examples: $gradu(\bar{r}) = grad\bar{r}^2 = \nabla(x^2 + y^2 + z^2) = 2x\vec{\imath} + 2y\vec{\jmath} + 2z\vec{k} = 2\vec{r}$. Laplace operator. $\Delta u = divgradu(\bar{r}) = \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$. Rules of derivation: $div\ (\bar{r}^2\vec{r}\ ) = 5\bar{r}^2$. Theoretical example: gravity $\vec{v}(\vec{r}) = gradu(\vec{r}),\ u(\vec{r}) = 1/|\vec{r}|$.

Week 7.

Lecture 13. (October 15, Tuesday).

Rotation (vector field): $\nabla \times \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v^1 & v^2 & v^3 \end{vmatrix}$.

Second order NABLA operators. Laplace operator. $\Delta u = divgradu(\bar{r}) = \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} +$

$\frac{\partial^2 u}{\partial z^2}$. $rotgradu(\bar{r}) = \nabla \times \nabla u(\vec{r}) = \vec{0}$, $div\ rot\ \vec{v}(\bar{r}) = \nabla \cdot (\nabla \times \vec{v}(\vec{r})) = \vec{0}$.

Introduction to potential theory. A vector field is potential, if there exists a scalar field such that its gradient is equal to the vector field - $gradu = \bar{v}$. Necessary condition: $rot\bar{v}(\bar{r}) = \bar{0}$. Example (trivial): $\bar{v}(\bar{r}) = 2\vec{r}$. $u(\bar{r}) = \bar{r}^2$.

Theoretical example – gravity. $\vec{v}(\vec{r}) = gradu(\vec{r}) = -\vec{r}/|\vec{r}|^3,\ u(\vec{r}) = 1/|\vec{r}|$.

Finding the potential function. Further (non-trivial) examples.

Cartesian coordinates. Find λ if the vector fields are potential. Determine the potential functions. $\vec{v}(\vec{r}) = (2x - 2z)\vec{i} + (2y + z)\vec{j} + (y - \lambda x)\vec{k}$.

Remarks Test 1. Viewing, results.

Information about Retake 1, October 30, 2024. The students whose mark of Test 1 was less than 15% or were absent must take part for the signature (there will be another, last Retake on December 10, 2024, registration in NEPTUN for this one will be necessary). If a student wants to improve his /her mark, he /she may take part at this retake (October 30). Registration at the attendance list is necessary. Please understand if you take part then the new mark will overwrite the old one. There is NO improvement on December 10, 2024.

There will be no lecture for other students.

Lecture 14. (October 16, 2024, Wednesday).

Finding the potential function. Further (non-trivial) examples.

Vector form. Find λ if the vector fields are potential. Determine the potential functions. $\vec{v}(\vec{r}) = \vec{r}^2\vec{k} + \lambda(\vec{k} \cdot \vec{r})\vec{r},\ \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

Definition of a curve (dimension 3, γ: $\bar{r}(t) = x(t)\bar{\imath} + y(t)\bar{\jmath} + z(t)\bar{k}, \left|\dot{\bar{r}}(t)\right| \neq 0,\ \alpha \leq t \leq \omega$)). Example: Straight line. Arc length of a curve. $l = \int_\alpha^\varpi \left|\dot{\bar{r}}(t)\right| dt$. Example: helix ($\bar{r}(t) = a\cos t\,\bar{\imath} + a\sin t\,\bar{\jmath} + bt\bar{k},\ 0 \leq t \leq 2\pi$). Arc length of a curve given as the graph of a function of 1 real variable $l = \int_a^b \sqrt{(f_x')^2 + 1}\,dx$. Example: circle.

Definition of line (curvilinear) integral $\oint_\gamma \vec{v}(\vec{r})d\vec{r} = \int_\alpha^\omega \vec{v}(\vec{r}(t))\dot{\vec{r}}(t)dt$. Work. Line integral over straight line, circle.

Week 8.

Lecture 15. (October 22, 2024, Tuesday).

Let $\bar{v} \in C^1(D), D \subset R^3$ a simply connected domain. The following 3 statements are equivalent:

1. $\bar{v}$ is potential (there exists a scalar field u such, that $gradu = \nabla u = \bar{v}$).
2. $\bar{v}$ is conservative (integral over any closed curve is 0) $\Leftrightarrow\ \int_\gamma \vec{v}(\vec{r})d\vec{r} = u(\vec{r}(\omega)) - u(\vec{r}(\alpha))$).
3. $\bar{v}$ is rotation free $rot\bar{v}(\bar{r}) = \nabla \times \bar{v}(\bar{r}) = \bar{0}$.

Definition of a simply connected region. Theoretical counterexample $u(\vec{r}) = arctan\frac{y}{x}$, $\vec{v}(\vec{r}) = gradu(\vec{r})$, integration over the unit circle.

Calculation of line integrals by definition and by the help of Potential theory in invariant and coordinate forms.

Theoretical example $\vec{v}(\vec{r}) = gradu(\vec{r}),\ u(\vec{r}) = 1/|\vec{r}|$.

Line integrals. Numerical exercises (in coordinates, in invariant form). Another – similar exercises were solved at the lecture.
Find the value of λ, if $\vec{v}(\vec{r}) = (2x - 3z)\vec{i} + 2(y + z)\vec{j} + (2y - \lambda x + 3z^2)\vec{k}$ is potential. Find a potential function. For this value of λ calculate the line integral along the segment connecting the points $A(0,1,0)$ and $B(1,1,3)$ by 2 methods (i) definition of the integral, (ii) by the help of the potential theory.
Homework (optional). Thomas Chapter 16. § 2,3,8. Exercises 3. Thomas Chapter 16. § 3. Exercises 15, 21.

Lecture 16. (October 23, 2024, Wednesday).

NO LECTURE. National holiday.

Week 9.

Lecture 17. (October 29, 2024, Tuesday).

Potential theory. Further example: Prove that the vector field is potential $\vec{v}(\vec{r}) = (\vec{j}\cdot\vec{r})(\vec{k}\cdot\vec{r})\vec{i} + (\vec{i}\cdot\vec{r})(\vec{k}\cdot\vec{r})\vec{j} + (\vec{i}\cdot\vec{r})(\vec{j}\cdot\vec{r})\vec{k},\ \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$. Find $\int_{\gamma_1}\vec{v}(\vec{r})d\vec{r}$ if

$\gamma_1 :\quad \vec{r}(t) = 4\cos t\,\vec{i} + 4\sin t\,\vec{j} + 3t\,\vec{k},\ 0 \le t \le \frac{\pi}{4}$.

Definition of a surface $F : \vec{r} = \vec{r}(u,v), \vec{r} \in C^1(T),\ \vec{r}_u \times \vec{r}_v \ne \vec{0}$. Surface area $\mu F = \iint_T |\vec{r}_u \times \vec{r}_v|\, dudv$. Example: Sphere - spherical coordinates.

Lecture 18. (October 30, 2024, Wednesday).

Retake 1. D. 816/B. The lecture starts at 8:15.

Rules of this retake (once more):
The students whose mark of Test 1 was less than 15% or were absent must take part for the signature (there will be another, last Retake on December 10, 2024, registration in NEPTUN for this one will be necessary). If a student wants to improve his /her mark, he /she may take part at this retake (October 30). Registration at the attendance list is necessary. Please understand if you take part then the new mark will overwrite the old one. There is NO improvement on December 10, 2024.
There will be no lecture for other students.

Week 10.

Lecture 19. (November 5, 2024, Tuesday).

Surface area in Cartesian coordinates. Surface area of a surface given as the graph of a function of 2 real variables $\mu F = \iint_D \sqrt{(f_x')^2 + f_y')^2 + 1}\ dxdy$. Example – semi-sphere.

Definition of the surface integral. $\iint_F \vec{v}(\vec{r}) \cdot d\vec{F} = \iint_T \vec{v}(\vec{r}(u,v)) \cdot (\vec{r}_u \times \vec{r}_v) dudv$. Meaning: flux.

Stokes' general theorem. Its special cases.

(i) Potential theory: $\int_\gamma \vec{v}(\vec{r}) d\vec{r} = \int_\gamma \nabla u = \int_{\partial\gamma} u = u(\vec{r}(\omega)) - u(\vec{r}(\alpha))$

(ii) Stokes special theorem. $\oint_\gamma \vec{v}(\vec{r}) d\vec{r} = \int_{\partial F} \vec{v} = \int_F \nabla \times \vec{v} = \iint_F rot\vec{v}(\vec{r}) d\vec{F}$.

(iii) Gauss-Ostrogradsky theorem: $\oiint_F \vec{v}(\vec{r})\, d\vec{F} = \int_{\partial V} \vec{v} = \int_V \nabla \cdot \vec{v} = \iiint_V div\vec{v} dV$.

Lecture 20 (November 6, Wednesday).

Gauss-Ostrogradsky theorem. $\oiint_F \vec{v}(\vec{r})\, d\vec{F} = \iiint_V div\vec{v} dV$.

Exercise. Calculate $\oiint_F \vec{v}(\vec{r}) d\vec{F}$ if $\vec{v}(\vec{r}) = \bar{r}$, F: $x^2 + y^2 + z^2 = 1$ unit sphere, normal points outward. By 3 methods (i) elementary, from the definition of surface integral, (ii) definition of surface integral, (iii) Gauss-Ostrogradsky theorem, divergence part.

Examples: $\vec{v}(\vec{r}) = (2x - z)\vec{\imath} + (y + z)\vec{\jmath} + z^2\vec{k}$. Calculate $\oiint_{Fi} \vec{v}(\vec{r}) d\vec{F}$, $i = 1, 2, 3$ by the Gauss-Ostrogradsky theorem, if the closed surface is the boundary of the domain given by the conditions (normal points outward): F_1: $x^2 + y^2 + z^2 \le 1$, $z \ge 0$, F_2 : $\sqrt{x^2 + y^2} \le z \le 1 + \sqrt{1 - x^2 - y^2}$, F_3 : $0 \le x \le 1$, $x^2 \le y \le \sqrt{x}$, $0 \le z \le 1$.

Gauss-Ostrogradsky theorem. $\oiint_F \vec{v}(\vec{r})\, d\vec{F} = \iiint_V div\vec{v} dV$. Consider the vector field $\vec{v}(\vec{r}) = (\vec{r} \times \vec{\imath})(\vec{r} \cdot \vec{\jmath})$, $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$. Calculate $\oiint_F \vec{v}(\vec{r}) d\vec{F}$ by the Gauss-Ostrogradsky theorem, if the closed surface is the boundary of the domain given by the conditions (normal points outward): F: $x^2 + y^2 \le 1$, $0 \le z \le 2$,

Homework (optional). Thomas. Chapter 16. §. 5., exercises 1, 3, 21.

Week 11.

Lecture 21 (November 12, Tuesday).

Stokes special theorem $\oint_\gamma \vec{v}(\vec{r}) d\vec{r} = \iint_F rot\vec{v}(\vec{r}) d\vec{F}$. Examples (i) Consider the vector field $\vec{v}(\vec{r}) = \vec{k} \times \vec{r}$. Find $\oint_\gamma \vec{v}(\vec{r}) d\vec{r}$, if γ is the boundary of the triangle $0 \le x \le 1$, $0 \le y \le -x + 1$, $z = 0$. Check the result by the help of Stokes theorem. (ii) Calculation of the integrals on both sides. Example: $\vec{v}(\vec{r}) = \vec{r}^2\vec{k}$. Find $\oint_{\gamma_1} \vec{v}(\vec{r}) d\vec{r}$, $\gamma_1 = \gamma_2 \cup \gamma_3 \cup \gamma_4$, where $\gamma_2 : x = 0,\ y = 2t,\ z = 0,\quad 0 \le t \le 1$, $\gamma_3 : x = 0,\ y = 2\cos t,\ z = 2\sin t,\ 0 \le t \le \frac{\pi}{2}$, $\gamma_4 : x = 0,\ y = 0,\ z = 2 - t,\quad 0 \le t \le 2$.

Lecture 22. (November 13, Wednesday).

Solution of a Sample Test 2.

Week 12.

Lecture 23. (November 19, Tuesday).
Formulation of Potential theory, Gauss-Ostrogradsky, Stokes theorems oi case $n = 2$.
Consultation before Test 2-
Solution of another sample test.

Lecture 24. (November 20, 8:15, Wednesday).
Test 1. Room D.316/B.

Week 13.

Lecture 25. (November 26, Tuesday).
Approximation of functions by power series (Maclaurin, Taylor). Example: Let $f(x) = e^{\frac{-x}{(D+1)}} \cdot x$. (i). Develop f into Maclaurin series until the 3rd order terms. (ii). Calculate an approximate value of $\int_0^1 f(x)\,dx$ by the help of the first order Maclaurin polynomial of f . Give an approximation of the error of this result by the integral of the second order term in the Maclaurin series. (iii). Calculate the exact value of this integral. (iv). Determine the value of $\lim\limits_{x\to+0} \frac{f(x)-\sin x}{x^a}$, $a = +1, +2, +3$. (v). Find the equation of the tangent line to the function $f(x) = e^{\frac{-x}{(D+1)}} \cdot x$ at the point $x_0 = \ln 1$.
Viewing of Test 2.

Lecture 26. (November 27, Wednesday).
Trigonometric polynomials, Fourier series. Example: Let $f(x) = \sin 2x \cdot \sin(x - 2\pi)x$. (Hint: before starting the solution of the exercise simplify the function by the help of trigonometric identities). (i) Develop f into Maclaurin series until the fourth order terms. (ii) Calculate $\int_0^{\frac{1}{2}} f(x)\,dx$ by the help of the first order Maclaurin polynomial of f . Give an approximation of the error of this result by the integral of the third term in the Maclaurin series. (iii) Write f in form of a trigonometric polynomial. (iv) Calculate the exact value of $\int_0^{\frac{1}{2}} f(x)\,dx$. (v) Calculate the limit $lim_{x\to 0} \frac{f(x)}{x^a}$ if $a = 1, 2$. (vi) Find the equation of the tangent line to the function f at the point $x_0 = 0$.
Extremum (local, global, conditional) problems. Example: Let $f(x,y) = (x^2 + y^2)e^{-x}$. (i) Find the stationary popints and their types. (ii) Determine the global extrema of f , if (x,y) belong ti $D = \{(x,y) | x^2 + y^2 \le 1\}$. (iii) Is there a global maximum, minimum, if $(x,y) \in R^2$?

Week 14. PLANNED

Lecture 27. (December 3, Tuesday)
Retake2 in R. 501.

Lecture 28. (December 4, Wednesday).
Viewing of Retake 2.
Sample Global (comprehensive) exam.