TEAMS. Mathematics EP2, Test 2. May 6, 2020.
*8:15 - 9:35. 80 minutes. 5*10=50%*
Good Luck!

Please **upload your results** in form of **one pdf** (prepared by CamScanner) until **9:45** as the latest.

The test depends on a parameter ***D***. Please start the **title of your uploaded work with this parameter *D* (1 number) and continue with your NEPTUN code**.

The parameter ***D*** is the **remainder** of the number of letters of **your full name (as in NEPTUN)** divided by **5**. E.g. I am in NEPTUN as Dr Moson Peter, the number of letters is 12. 12=2*5+2, so *D=2*. Please **substitute** your ***D*** in the test and carry out the calculation with this value.

Test is on the next page.

1. Consider the function $f(x,y) = -(x^2+y^2)e^{(D+1)x}$ and the domains $T_1 = \{(x,y) \mid x^2+y^2 \le (5-D)^2,\ \ 0 \le y\}$, $T_2 = \{(x,y) \mid 0 \le x \le 3,\ 0 \le y \le 3D+3\}$. Sketch the domains.
(i) Find the stationary points of f, determine their type (minimum, saddle, etc.).
(ii) Find the global extrema (maximum-minimum points and values) of f on T_1.
(iii) Find the center of gravity (mass) of homogeneous domains T_1, T_2 (the density is $\rho(x,y) \equiv 1$.).
(iv) Find the moment of inertia of homogeneous domain T_2 (the density is $\rho(x,y) \equiv \frac{1}{54}$).
2. Find the closest, farthest points (if they exist) of the set $T_3 = \{(x,y) \mid x+y = 2D\}$ to the point $P = (6,6)$ by 2 methods (some possible methods: elementary, conditional extremum, parametrization of the boundary). What is the value of these distances?

SOLUTIONS

1. Semicircle, rectangle.
(i) $(x_1,y_1) = (0,0)$ maximum. $(x_2,y_2) = \left(\frac{-2}{D+1}, 0\right)$ saddle. *10%*
(ii) Minimum $-(5-D)^2 e^{(D+1)(5-D)}$ at $(5-D,0)$. Maximum 0 at $(0,0)$. *10%*

(iii) The center of gravity of $T_1 : (x_c,y_c) = \left(0, \frac{(5-D)4}{3\pi}\right)$, $T_2 : (x_c,y_c) = \left(\frac{3}{2}, \frac{(3D+3)}{2}\right)$. *8+2=10%*

(iv) The moment of inertia of T_2: $I_0 = ((D+1) + (D+1)^3)/2$. *10%*

2. Closest $Q = (D, D)$, distance $(6 - D)\sqrt{2}$. There is no farthest point.
4+4+2=10%