Mathematics A3. Test 1. October 13, 2020 at 8:15. a.m. Room MGFEA.

Exercises. 50%=10+8+8+14+10%

1. Find the solution of the initial value problem $y' = 2x + y, y(0) = 0$. Check it. *7+2+1=10%*

2. Sketch the one-dimensional phase portrait of the autonomous equation $y' = -4y + y^3$. Find the inflection points of the solutions and sketch some integral curves in the plane. *4+4=8%*

3. Find the orthogonal trajectories of the curves $y = c(x+2)^{-1}$. Sketch both family of curves. What kind of curves (circle, parabola, etc.) are the original and the orthogonal ones? *4+4=8%*

4. Solve the initial value problem $y(0) = 0$, $y'(0) = 3$ for the second order equation $y'' - 2y' - 3y = 3$ by 2 methods: (i) linear equation with constant coefficients, (ii) Newton' method until the 3rd order terms. Sketch the graph of the solution. *8+4+2=14%*

5. Consider the autonomous system $\dot{x} = -5y,\ \dot{y} = 5x$. Find the general solution, sketch the phase portrait. 5+5=*10%*

Solutions (without figures)

1. Linear equation. General solution: $y = ce^x - 2(x+1)$. The solution of the initial value problem $y(0) = 0$ is $y = 2e^x - 2(x+1)$. *7+2+1=10%*

2. The stationary point $y_1 = 0$ is stable, $y_{2,3} = \pm 2$ are unstable. The inflection points of the integral curves are $y_{4,5} = \pm\sqrt{\frac{4}{3}}$. *4+4=8%*

3. The differential equation of the original family $y' = \frac{-y}{x+2}$, of the orthogonal family $y' = \frac{x+2}{-y}$. The orthogonal trajectories are hyperbolas $\frac{(x+2)^2}{2} + \left(\frac{-1}{2}\right) y^2 = c^2$. *4+4=8%*

4. General solution $y = c_1 e^{-x} + c_2 e^{3x} - 1$. Initial value problem $y = e^{3x} - 1 = 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + \cdots$ *(6+2)+4+2=14%*

5. The stationary point is a center ($\lambda_{1,2} = \pm 5i$), the trajectories are circles (orientation is counterclockwise), the general solution is $x(t) = c^1 \cos 5\,t + c^2 \sin 5\,t$, $y(t) = c^1 \sin 5\,t - c^2 \cos 5\,t$. *10%*