MATH EP1. TEST 2. November 25, 2020, 12:15 p.m.

90 minutes. 20%. *Good Luck!*

Assignment part of the TEAM.

Please **upload your results** in form of **one pdf** (prepared by CamScanner) until **13:55** as the latest.

The test depends on a parameter ***D***. Please start the **title of your uploaded work with this parameter *D* (1 number) and continue with your NEPTUN code**.

The parameter ***D*** is the number of letters **e, p, k** in **your full name (as in NEPTUN)** divided by ***7***.

E.g. I am in NEPTUN as Dr Moson Peter. There is no **k** in my full name, there are 2 **e** and 1 **p**. Altogether the number of letters **e, p, k** is 2+1+0=3. 3=0*7+3, so my ***D=3***.

Please **substitute** your ***D*** in the test and carry out the calculation with this value.

Test is on the next page.

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1.Consider the homogeneous ($\rho(x,y) \equiv 1$) planar domain $T = \left\{(x,y) \,\middle|\, +1 \le x \le +2, \quad 0 \le y \le \frac{D+1}{x}\right\}$. Find the coordinates of the center of mass (gravity). 7%

2. Consider the function $f(x) = (7 - D)\sin(x^2)$. (i) Develop it into Maclaurin series. (ii) Find the value of $(7 - \mathrm{D})\sin(-\frac{1}{2})$ by the help of the 3rd order Maclaurin polynomial of f. Estimate the error. (iii) Calculate $\int_0^{0,5} f(x)\,dx$ with 2 decimal digits (error $0{,}005$). (iv) Determine the value of $\lim\limits_{x\to 0} \frac{f(x)-(7-D)x^{2a}}{x^6}$ $, a = +1, +2$. *6%*

3. Calculate the following indefinite integrals (i) $\int \frac{x^5+D}{x^3+x}\,dx$, (ii) $\int x \ln((7 - D)x)dx$. *7%*

SOLUTIONS (without figures)

1. $m=(D+1)\ln 2$, $x_c=\frac{1}{\ln 2}$, $y_c=\frac{D+1}{4\,ln\,2}$. *2+1+2+2=7%*
2. (i). $f(x)=(7-\mathrm{D})\sin(x^2)=(7-\mathrm{D})\,(x^2-\frac{x^6}{6}+\ldots)$. (ii) $(7-\mathrm{D})\sin(-\frac{1}{2})=-f\left(\frac{1}{\sqrt{2}}\right)=(7-\mathrm{D})\left(-\left(\frac{1}{\sqrt{2}}\right)^2+\frac{\left(\frac{1}{\sqrt{2}}\right)^6}{6}+\ldots\right)=(7-\mathrm{D})(-\frac{1}{2})$. Error < $\frac{(7-\mathrm{D})}{48}$. (iii) $\int_0^{\frac{1}{2}} f(x)\,dx=(7-\mathrm{D})\left[\frac{x^3}{3}-\frac{x^7}{42}+\ldots\right]_0^{\frac{1}{2}}\approx\frac{(7-\mathrm{D})}{24}$. (iv) $l=\lim_{x\to 0}\frac{f(x)-(7-\mathrm{D})x^{2a}}{x^6}$. If $a=1\ \to l=-\frac{(7-\mathrm{D})}{6}$, if $a=+2\ \to\ l=+\infty$.
   2+1+1,5+(1+0,5)=6%.
3. (i) $\frac{x^5+D}{x^3+x}=x^2-1+\frac{x+D}{x(x^2+1)}=x^2-1-\frac{Dx}{(x^2+1)}+\frac{D}{x}+\frac{1}{(x^2+1)}$. $\int\frac{x^5+D}{x^3+x}dx=\frac{x^3}{3}-x-\frac{D}{2}\ln(x^2+1)+D\ln x+\arctan x+c$. *4%*
   $(ii)\int x\ln((7-D)x)dx=\int x\ln(7-D)dx+\int x\ln x dx$.
   $\int x\ln((7-D)x)dx=\ln(7-D)\frac{x^2}{2}+\frac{x^2}{2}\ln x-\frac{x^2}{4}+c$.
   Integration by parts. *3%* (ii)