Mathematics EP2. TEST 1. 2020-03-25. 8:15-10:00. TEAMS. 80 minutes. 15+15+10+10=50%. =50%. *Good Luck!*

You can solve it on paper (or with more sophisticated technics).
Write your name and Neptun code on all paper you turn in.
Take a photo of your solution (or with more sophisticated technics) and turn it in in TEAMS.

The parameter ***D*** you can see below is the last digit of your birthday. For example if your birthday is September 2**3**, then ***D*=*3***. Substitute your value of ***D*** instead of all ***D***-s in the exercises and solve them.
Do not forget to write at the beginning of your answer the value of your ***D*.**

The Test is on the next page.

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1. Consider the complex numbers: $z_1 = \sqrt{3} + Di$, $z_2 = (D+1)\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$. (i) Find the algebraic form of z_2. Calculate the value of the following complex numbers: (ii) $z_1 - z_2$, (iii) $z_1 \cdot z_2$, (iv) $z_2^{\,7-D}$. *3*3+6=15%*

2. Find the solution $y = y(x)$ of the initial value problem $y'' + 2y' - 3y = D$, $y(0) = \frac{3-D}{3}$, $y'(0) = -3$ (by 2 methods: linear equation, Newton's method until the 3rd order terms). *10+5=15%*

3. Find the general solution of the equation $y'' - 2y' + 2y = Dx$. *2*4=10%*

4. Consider the autonomous system of ODEs $\dot{x} = -(D+1)y,\ \dot{y} = (D+1)x + 5y$. Determine the type of the equilibrium point (node, saddle, etc.), and sketch the phase portrait. Find the general solution as well. *10%*

SOLUTIONS

1. (i). $z_2 = D\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = -Di$ (ii) $z_1 - z_2 = \sqrt{3} + (2D+1)i$, (iii) $z_1 \cdot z_2 = --D(D+1) - i(D+1)\sqrt{3}$ (iv) $z_1^{\,6} = 2^6\left(\cos 6\cdot\frac{\pi}{3} + i\sin 6\cdot\frac{\pi}{3}\right) = 64$, (iv) $z_2^{\,4} = 16$.

*5*2=10%*

2. The general solution $x = c_1e^x + c_2e^{-3x} - \frac{D}{3}$. The solution of the solution of the initial value problem is $y = e^{-3x} - \frac{D}{3} = \frac{3-D}{3} - 3x + \frac{9}{2}x^2 - \frac{9}{2}x^3 + \ldots$. *10+5=15%*

3. The general solution $y = c_1e^x\cos x + c_2e^x\sin x + \frac{D}{2}(x+1)$. *10%*

4. The characteristic equation $\lambda^2 - 5\lambda + 4(D+1)^2 = 0$. The discriminant is $25 - 4(D+1)^2$. If *D*=1,2, then the phase portrait is unstable node. If D>2 then it is an unstable focus. E.g. for D=1 $\lambda_1 = 1, \lambda_2 = 4$. The general solution is $x(t) = -2c^1e^{-t} + c^2e^{-4t}$, $y(t) = c^1e^{-t} - 2c^2e^{-4t}$. *10%*