Detailed program (planned) of the course

DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS[1], [2]

Week 1.

Lecture

1. Systems of ordinary differential equations (ODE). Initial value problem, Theorem of existence and uniqueness. Gronwall lemma (I. Bihari). Proof of uniqueness.
2. First order ordinary differential equations. Some methods of solution (integration, geometrical – isoclines, idea of Newton, undetermined coefficients method, idea of the Euler method). Separable equations.
3. Short test (to have some information about the knowledge of the students –collected, but without grades and detailed correction). Solving the exercises of the test.

Tutorial (practical lecture)

1. First order ordinary differential equations. Some methods of solution (integration, computer algebra, series, explicit / implicit Euler, Runge-Kutta methods) demonstrated on separable equations.

Week 2.

Lecture

1. Systems of linear ordinary differential equations (ODE). Short summary of the theory (general, constant, periodic coefficients cases).
2. Dependence on initial conditions, parameters. Linearization. Variational system.
3. Example (model of the spaceship, F. Brenn – V. Beletzky).

Tutorial

1. First order ordinary differential equations. Some methods of solution (integration, computer algebra, series, explicit / implicit Euler, Runge-Kutta methods) demonstrated on linear equations.

Week 3.

Lecture

1. Linear (first and higher order) ordinary differential equations. Short summary of the general theory.
2. Theorem of structure. General result for linear problems.
3. Special linear second and higher order equations (with constant coefficients, Euler, Bessel).
4. Mechanical models, oscillation (A. Elbert).
5. Short test (graded, but only for the information of the students). Solving the exercises of the test. Some more exercises.

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[1] Code of the subject: BMETE90MX46, Contact hours: 4 lectures + 2 tutorials + 0 lab / week, Credit: 8, Evaluation: exam, Prerequisites: BSc diploma, Semester: 2nd (in the academic year), 1st (in the given MSc program)

[2] The lectures will speak about the related results of late Hungarian mathematicians (among others former profs of BME) as well. Their names are written in blue.

Tutorial

1. Second order ordinary linear equations. Analytic and numerical solution.

Week 4.

Lecture

1. Autonomous systems. 1- dimensional case.
2. 2-dimensional autonomous systems. Linearization . Phase space analysis near equilibrium points (linearization, Poincaré theory), periodic orbits.
3. Examples (population dynamics, predator-prey equation, M. Farkas).

Tutorial

1. Sketching phase portraits of 2, 3 dimensional autonomous systems.

Week 5.

Lecture

1. The loss of stability in parameterized families of equations.
2. 1 co-dimensional bifurcations (Saddle-Node, Hopf).
3. Examples (robustness).

Tutorial

1. Investigation of bifurcation problems

Week 6.

Lecture

1. Different stability concepts, definitions, examples.
2. Lyapunov functions. Lyapunov's theorems (stability, asymptotic stability, instability).
3. Mechanical examples.

Tutorial

1. Lyapunov functions. Estimation of the domains of stability.

Week 7.

Lecture

1. Introduction to functional analysis (topological, metric, Banach, Hilbert spaces).
2. **Test 1** (graded on scale 0-20%).

Tutorial

1. Comparing exact and approximate dynamics, error estimate between exact and approximate solutions.

Week 8.

Lecture

1. Lyapunov stability by the first approximation (linearization). Routh-Hurwitz criterion.

2. Example (model of the steam engine).
3. Solving numerical examples.

Repeated Test 1 (for students absolving the original one with a grade less than 6% - the date will be fixed later).

Tutorial

1. Comparing exact and approximate dynamics, error estimate between exact and approximate solutions.

Week 9.

Lecture

1. Hilbert spaces. Fourier series.
2. Convergence of trigonometric series (point wise, $C_{2\pi}$, $L_{2\pi}$ - F. Riesz, L. Fejer)

Tutorial

1. Exercises. Expanding functions into Fourier series.

Week 10.

Lecture

1. Partial differential equations. Classification.
2. Heat transfer equation in Cartesian and in cylindrical coordinates. Fourier method.

Tutorial

1. The method of finite differences for the heat transfer equation, error estimate, the maximum principle.

Week 11.

Lecture

1. Sturm-Liouville problems. Sobolev spaces.
2. Wave equation (Fourier, D'Alembert methods).

Tutorial

1. Numerical solution of partial differential equations.

Week 12.

Lecture

1. Some other topics (retarded equations, dynamical systems etc.).
2. **Test 2** (graded on scale 0-20%).

Tutorial

1. Numerical solution of equations.

Weeks 13-14.

Lecture. Additional topics by the choice of the students. They can be:

1. Proof of some theorems (e.g. Theorem of existence and uniqueness, Lyapunov by the first approximation).
2. Introduction to some (non considered) theories (e.g. Calculus of variations).
3. Introduction to some (non considered) methods of solution (e. g. Laplace transformation).

Tutorial. Discussion of the home works. Students' presentations.

Repeated Test 2 (for students absolving the original one with a grade less than 6% - the date will be fixed later).

Weeks 15.

Repeated test for students with one test with grade less than 6%. Late fee is applied. Planned date: May 20, 2014.

Budapest, January 16, 2014

Lectures: Peter Moson
Tutorials (practical lectures) Miklos Mincsovics