DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS[1]

Questions – Oral Exam. Spring 2014

1. Systems of ordinary differential equations (ODE). Integral curves, phase portraits (in case of autonomous systems). Initial value problem, Theorem of existence and uniqueness (only formulation).
2. Lipschitz condition. Gronwall lemma. Proof of uniqueness, continuous dependence on initial conditions.
3. First order separable ODEs. Phase portraits of one dimensional autonomous systems.
4. Systems of linear ordinary differential equations. General case. The maximal interval of solutions coincides with the domain of definition with respect to t of the right hand side (proof by Gronwall lemma).
5. Homogeneous linear systems (H). Fundamental matrix, system. Wronski determinant. Non-homogeneous systems (IH). Method of variation of constants. Theorem of Structure (General solution of (IH) = General solution of (H) + Particular solution of (IH)).
6. First order linear ODE. Maximal interval, formula, structure of general solution.
7. Linear systems with constant coefficients. General solution (by 2 methods: (i) exponential matrix function, eigenvalues, eigenvectors).
8. Phase portraits in case of 2 dimensional linear systems with constant coefficients.
9. Second order ODEs. Short summary of the general theory. Reducible equations. Initial, boundary value problems.
10. Linear second order equations with constant coefficients.
11. Laplace transformation. Definition. Application to differential equations, systems.
12. Lyapunov stability by the first approximation (linearization). Proof in case of asymptotic stability (based on Gronwall lemma). Routh-Hurwitz criterion.
13. Differentiable dependence on initial conditions, parameters. Linearization. Variational system.
14. 2-dimensional autonomous systems. Phase space analysis near equilibrium points (linearization, Poincaré theory).
15. Typical 1-codimensional bifurcations. Saddle-node bifurcation.
16. Typical 1-codimensional bifurcations. Andronov-Hopf bifurcation.
17. Lyapunov functions. Lyapunov stability theorem (with proof).
18. Lyapunov functions. Asymptotic stability, instability. Stability of linear second order equations.
19. Definition of topological, metric spaces. Banach fixed point theorem.
20. Definition of Banach, Hilbert spaces. C^o, L^2 spaces.
21. Fourier series. Convergence.
22. Fourier method for heat transfer equation.
23. Application. Asymptotic stability of the centrifugal regulator (Watt).
24. Application (differential dependence on initial conditions). Space ship model
25. * .

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[1] Code of the subject: BMETE90MX46, Contact hours: 4 lectures + 2 tutorials + 0 lab / week, Credit: 8, Evaluation: exam,