Mathematics EP1 Exam, BME. 2019-12-17. 10:00. E1A. 90 minutes. *Good Luck*

<u>Theory</u> *(3*3=9%.). Maximum 15 minutes.* Name 3 scientists, who had important mathematical research in the theory of derivation, integration of functions of one real variable. Write the main idea of the mathematical result (definition, theorem, etc.).

<u>Exercises</u> 12+10+10+19=51%. Minimum 75 minutes. Explain your solution. The use of paper based documents is allowed.

1. Calculate the coordinates of the center of gravity of the homogeneous ($\rho(x,y)\equiv 1$) domain $D=\left\{(x,y)\,\middle|\, -\frac{\pi}{4}\le x\le +\frac{\pi}{4},\ 0\le y\le \cos x\right\}$. *12%*
2. Let $f(x)=\sin 2x\cdot\cos x$. (i) Find the 3rd order Maclaurin polynomial (Taylor at $x_0=0$) of the function f . (ii) Calculate the limit $\lim_{x\to 0}\frac{f(x)-2x}{x^3}$. *10%*
3. $OA=\vec a=2\vec i-2\vec j-\vec k=(2,-2,-1)$, $OB=\vec b=\vec i+\vec j=(1,1,0)$. Find $\vec a+\vec b,\ \vec a\cdot\vec b,\ \vec a\times\vec b$. How much is the perimeter, the area of the triangle OAB ? *10%*
4. Find the general solution of the differential equation $y'=\frac{y}{x+2}+2x^2+4x,\ -2<x$. Solve the initial value problem $y(0)=-2$. Sketch the graph of this solution (roots, stationary – inflection points, etc.). *19%*

Solutions (without figures).

<u>Theory.</u> Name 1%. Name and approx. result. 2%, Name and exact result 3%. Total *3*3=9%.*

<u>Exercises</u>

1. $x_c=0$ (symmetry). $m=\int_{-\frac{\pi}{4}}^{+\frac{\pi}{4}}\cos x\,dx=[\sin x]_{-\frac{\pi}{4}}^{+\frac{\pi}{4}}=\sqrt{2}$. $\frac{1}{2}\int_{-\frac{\pi}{4}}^{+\frac{\pi}{4}}\cos^2 x\,dx=\frac{1}{2}\left[\frac{x}{2}+\frac{\sin 2x}{4}\right]_{-\frac{\pi}{4}}^{+\frac{\pi}{4}}=\frac{\pi+2}{8}$.

$y_c=\frac{\pi+2}{8\sqrt{2}}$. *12%.*

2. (i) $f(x)=\sin 2x\cdot\cos x=(2x+\frac{4}{3}x^3+\dots)\cdot\left(1-\frac{x^2}{2}+\dots\right)=2x-\frac{7}{3}x^3+\dots$. (ii) $\lim_{x\to 0}\frac{f(x)-2x}{x^3}=-\frac{7}{3}$.

10%

3. $\vec a+\vec b=(3,-1,-1),\ \vec a\cdot\vec b=0,\ \vec a\times\vec b=(1,-1,4)$. *Perimeter* $=3+\sqrt{2}+\sqrt{11}$, *Area* $=\frac{3\sqrt{2}}{2}$. *10%*

4. Linear equation. General solution: $y(x)=c(x+2)+(x+2)x^2$. Solution of the initial value problem: $y(x)=(x+2)(x^2-1)=(x+2)(x+1)(x-1),\ -2<x$. Roots $x_{1,2}=\pm 1,\ -2<x$, maximum $x_3=\frac{-1-\sqrt{7}}{3}$, minimum $x_4=\frac{-1+\sqrt{7}}{3}$, inflection point $x_5=-\frac{2}{3}$, no asymptote. *Total 19%.*