Mathematics EP1 Exam, BME. 2019-01-21. 10:00. E1A. 90 minutes. *Good Luck*

*Theory (4*2,5=10%). Maximum 15 minutes.* arctan (inverse tangent) function. Sketch its graph, give its derivative, integral, Maclaurin series (until 2nd order terms).

Exercises 14+10+10+16=50%. Minimum 75 minutes. Explain your solution. The use of paper based documents is allowed.

1. Calculate the coordinates of the center of gravity of the homogeneous ($\rho(x,y)\equiv 1$) domain $D=\{(x,y) \mid 0\le x\le +1,\ 0\le y\le e^x\}$. Sketch the domain. *14%*
2. Let $f(x)=xe^x-x$. (i) Find the 3rd order Maclaurin polynomial (Taylor at $x_0=0$) of the function f. (ii) Calculate the limit (if it exists) $\lim_{x\to 0}\frac{f(x)}{x^k}$, *for* $k=1,2,3$. *10%*
3. $OA=\vec{a}=\frac{\vec{i}+2\vec{j}+2\vec{k}}{3}=\left(+\frac{1}{3},+\frac{2}{3},+\frac{2}{3}\right)$, $OB=\vec{b}=-\vec{i}-2\vec{j}+2\vec{k}=(-1,-2,+2)$. Find the length of vectors $\vec{a}$, $\vec{b}$. Calculate $|\vec{a}-\vec{b}|, |\vec{a}\cdot\vec{b}|, |\vec{a}\times\vec{b}|$. Give some geometric meaning of the previous results. *10%*
4. Find the general solution of the differential equation $y'=y+e^x$. Solve the initial value problem $y(0)=1$. Sketch the graph of this solutions (roots, stationary – inflection points, etc.). *16%*

Solutions (without figures).

Theory. Graph, derivative, integral, Maclaurin series 2,5%. Total *4*2,5=10%.*

Exercises

1. $m=\int_0^1 e^x dx=e-1$. $\int_0^1 xe^x dx=\left[(x-1)e^x\right]_0^1=1$. $x_c=\frac{1}{e-1}$. $\frac{1}{2}\int_0^1 e^{2x}dx=\frac{1}{2}\left[\frac{e^{2x}}{2}\right]_0^1=\frac{e^2-1}{4}$ $y_c=\frac{\frac{e^2-1}{4}}{e-1}=\frac{e+1}{4}$. *14%.*

2. (i) $f(x)=xe^x-x=x\left(1+x+\frac{x^2}{2}+...\right)-x=x^2+\frac{x^3}{2}+...$. (ii)

$k=1$ $\lim_{x\to 0}\frac{x^2+\frac{x^3}{2}+...}{x^1}=\lim_{x\to 0}(x+\frac{x^2}{2}+...=0$,

$k=2$ $\lim_{x\to 0}\frac{x^2+\frac{x^3}{2}+...}{x^2}=\lim_{x\to 0}(1+\frac{x^2}{2}+...)=1$. If $k=3$ then there is no limit. *10%*

3. $|\vec{a}|=1, |\vec{b}|=3$. $|\vec{a}-\vec{b}|=\left|\left(+\frac{4}{3},+\frac{8}{3},-\frac{4}{3}\right)\right|=\sqrt{\frac{32}{3}}, |\vec{a}\cdot\vec{b}|=\left|\frac{-1}{3}\right|=\frac{1}{3}, |\vec{a}\times\vec{b}|=\left|\left(\frac{8}{3},\frac{4}{3},0\right)\right|=\sqrt{\frac{16}{3}}$. $|\vec{a}-\vec{b}|$ is the length of side AB of the triangle OAB. $|\vec{a}\cdot\vec{b}|$ is the length of projection of vector $\vec{b}$ on vector $\vec{a}$. $|\vec{a}\times\vec{b}|/2$ is the area of triangle OAB. *10%*

4. Linear equation. The general solution $y(x)=ce^x+xe^x$

The solution of the initial value problem $y(0)=1$ is $y(x)=e^x+xe^x=(x+1)e^x$.

Root $x=-1$, minimum $x=-2$, inflection point $x=-3$, asymptote $y=0$ when $x\to-\infty$. *Total 8+2+6=16%.*