Mathematics EP1 Exam, BME. 2019-01-07. 10:00. E1A. 90 minutes. *Good Luck*

<u>Theory</u> *(3*3=9%.). Maximum 15 minutes.* Define the cosine function by different methods (at least three). Sketch its graph, give its derivative, integral.

<u>Exercises</u> *12+10+10+19=51%. Minimum 75 minutes. Explain your solution. The use of paper based documents is allowed.*

1. Calculate the coordinates of the center of gravity of the homogeneous ($\rho(x,y)\equiv 1$) domain $D=\{(x,y)\,|\,-2\le x\le +2,\ 0\le y\le 8-|x^3|\}$. Sketch the domain. *12%*
2. Let $f(x)=e^{2x}\cdot\cos x$. (i) Find the 3rd order Maclaurin polynomial (Taylor at $x_0=0$) of the function f . (ii) Calculate the limit (if it exists) $\lim_{x\to 0}\dfrac{f(x)-\sin\frac{\pi}{2}}{x^k}$, *for* $k=0,1,2$. *10%*
3. $OA=\vec{a}=\vec{i}+2\vec{j}+3\vec{k}=(+1,+2,+3)$, $OB=\vec{b}=-2\vec{i}-2\vec{j}+2\vec{k}=(-2,-2,+2)$. Find $\vec{a}+\vec{b}$, $\vec{a}\cdot\vec{b}$, $\vec{a}\times\vec{b}$. How much is the perimeter, the area of the triangle OAB ? *10%*
4. Find the general solution of the differential equation $y'=2xy^2-4xy+x\sqrt[3]{8}$. Solve the initial value problems (i) $y(0)=0$, (ii) $y(0)=1$. Sketch the graphs of these solutions (roots, stationary – inflection points, etc.) in one coordinate system. *19%*

Solutions (without figures).

<u>Theory.</u> Each definition 2%. Graph, derivative, integral 1%. Total *3*2+3*1=9%.*

<u>Exercises</u>

1. $x_c=0$ (symmetry). $m=2\int_0^2(8-x^3)dx=\left[16x-\frac{x^4}{2}\right]_0^2=24$. $\int_0^2(8-x^3)^2dx=\left[64x-4x^4+\frac{x^7}{7}\right]_0^2=\frac{576}{7}$.

$y_c=\frac{24}{7}\approx 3{,}4$. *12%.*

2. (i) $f(x)=e^{2x}\cdot\cos x=1+2x+\frac{3}{2}x^2+\frac{1}{3}x^3+\ldots$. (ii)

$k=0\quad \lim_{x\to 0}\dfrac{f(x)-\sin\frac{\pi}{2}}{x^k}=\lim_{x\to 0}\dfrac{1+2x+\ldots-1}{1}=0$,

$k=1\quad \lim_{x\to 0}\dfrac{f(x)-\sin\frac{\pi}{2}}{x^k}=\lim_{x\to 0}\dfrac{1+2x+\ldots-1}{x}=2$. if $k=2$ then there is no limit. *10%*

3. $\vec{a}+\vec{b}=(-1,0,5)$, $\vec{a}\cdot\vec{b}=0$, $\vec{a}\times\vec{b}=(10,-8,2)$. *Perimeter* $=\sqrt{14}+\sqrt{8}+\sqrt{26}$, *Area* $=\sqrt{42}$. *10%*

4. Separable equation.

The solution of the initial value problem $y(0)=1$ is $y(x)=1$.

The solution of the initial value problem $y(0)=0$ is $y(x)=1-\frac{1}{1+x^2}=\frac{x^2}{1+x^2}$.

Root $x=0$, minimum $x=0$, inflection point $x=\pm\frac{1}{\sqrt{3}}$, asymptote $y=1$. *Total 19%.*