MATHEMATICS A3, Retake 2. December 10, 2019. *R. 513..* **80 minutes.** 50% - *Good Luck!*

Consider the vector field $\vec{v}(\vec{r}) = \lambda\,(\vec{r}\times\vec{r}) + 2\left(3y^2+z\right)\vec{j} + (\lambda y + 2z)\vec{k}$, $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

(i) Find λ such that the vector field is potential. For this value of λ calculate the line integral along the segment connecting the points *(-2,0,3)* and *(1,2,3*) by 2 methods (i) definition of the integral, (ii) by the help of the potential theory. *18%.*

(ii) For $\lambda = 0$ find $\oint_\gamma \vec{v}(\vec{r})d\vec{r}$, if γ is the boundary of the ellipse (orientation is counterclockwise from the direction of the x-axis) $\frac{y^2}{4} + z^2 \le 1,\ \ x = 0$. Check the result by the help of Stokes theorem. *15%.*

(iii) For $\lambda = 0$ calculate $\oiint_{F_i} \vec{v}(\vec{r})d\vec{F}$, $i = 1,2,3$ by the Gauss-Ostrogradsky theorem, if the closed surfaces are the boundaries of the domains given by the conditions (normal points outward): V_1: $x^2 \le 1,\ y^2 \le 1, z^2 \le 1$, $V_2 : x^2 + y^2 \le 1,\ z^2 \le 1$, $V_3 : x^2 + y^2 + z^2 \le 1$ Sketch the domains. *17%.*

Solutions (without figures)

1. (i) $\vec{v}(\vec{r}) = \left(6y^2 + 2z\right)\vec{j} + (\lambda y + 2z)\vec{k}$ is a potential vector field if $\lambda = 2$. *4%.* $u(x,y,z) = 2y^3 + 2yz + z^2$. $\int_\gamma \vec{v}(\vec{r})d\vec{r} = u(B) - u(A) = 28$. *7%.* $\int_\gamma \vec{v}(\vec{r})d\vec{r} = 28$ by definition as well *7%. Total 18%.*

(ii) $\vec{v}(\vec{r}) = \left(6y^2 + 2z\right)\vec{j} + 2z\,\vec{k}$. $rot\vec{v}(\vec{r}) = -2\vec{i}$. $\oint_\gamma \vec{v}(\vec{r})d\vec{r} = -4\pi$. *3+2*6=15%.*

2. $\vec{v}(\vec{r}) = \left(6y^2 + 2z\right)\vec{j} + 2z\,\vec{k}$. $div\vec{v} = 2 + 12y.$. *2%* . $\oiint_{F_1} \vec{v}(\vec{r})d\vec{F} = 16$. *5%* . $\oiint_{F_2} \vec{v}(\vec{r})d\vec{F} = 4\pi$. *5%* . $\oiint_{F_3} \vec{v}(\vec{r})d\vec{F} = 8\pi/3$. *5%. Total 17%.*