BME Mathematics A2. Part Calculus. Exam May 29, 2018. 12:00. StMax. 40%. 1 hour.

<u>*Theory*</u> *(2*3=6%.). Maximum 10 minutes.* 1. Definition of spherical coordinates. Jacobi determinant. Substitution in triple integrals. 2. Fourier series. Definition. Formula for the coefficients.

<u>*Exercises*</u> *8+12+14=34%. Minimum 50 minutes.*

1. Consider the function $f(x,y)=1-x^2-y^2$. Find the stationary points of and determine their type (maximum, minimum, saddle, etc.). Find r, if the value of the double integral $\iint_D f(x,y)dxdy$ is maximal, where $D=\{(x,y) \mid x^2+y^2 \le r\}$. *8%*

2. Consider the function $f(x)=\sin x \cdot \sin 3x$. (i) Write the function f in form of a trigonometric polynomial. (ii) Calculate the value of the integral $\int_0^{\frac{\pi}{2}} f(x)dx$. (iii) Find the 4th order Maclaurin polynomial (Taylor at $x_0=0$) of the function f. (iv) Calculate the limit $\lim_{x\to 0}\frac{f(x)}{x^2}$. *11%*

3. Sketch following domains $D_1=\{(x,y,z) \mid x^2+y^2+z^2 \le 1\}$, $D_2=\{(x,y,z) \mid x^2+y^2 \le 1, -1 \le z \le 1\}$. Calculate the value of the triple integrals $\iiint_{D_i}(6x+2)dxdydz$, $i=1,2$. *3*5=15%*

Solutions

1. Maximum at $(x_1,y_1)=(0,0)$. $r=1$. *4*2=8%*
2. (i) $f(x)=\sin x\cdot\sin 3x=\frac{1}{2}\cos 2x-\frac{1}{2}\cos 4x$. (ii) $\int_0^{\frac{\pi}{2}} f(x)dx=0$. (iii) $f(x)=\sin x\cdot\sin 3x=\frac{1}{2}\cos 2x-\frac{1}{2}\cos 4x=3x^2-5x^4+\ldots$, (iv) $\lim_{x\to 0}\frac{f(x)}{x^2}=3$.
   *4*3=12%*
3. $D_1:\frac{8\pi}{3},\quad D_2: 4\pi$. *2*7=14%*