Differential Equations 1. October 10, 2018. 90 minutes. 17%.

Good Luck!

1. Sketch the one dimensional phase portrait of the autonomous equation $y' = \frac{1}{2} - \cos^2 y$, $-\pi \le y \le +\pi$. Find the inflection points of the solutions and sketch some integral curves in the plane. *1+1,5=2,5%*

2. Find the orthogonal trajectories of the curves $x^2 + y^2 = 2(2x + y)) + c$. Sketch both family of curves. *1+1=2%*

3. Find the solutions of initial value problems $y(0) = 1$, $y(0) = \frac{1}{2}$ and $y(1) = 0$ for equation $y' \cdot e^x = y^2$. Sketch their graphs. . *2+0,5+0,5+0,5=3,5%*

4. Consider the equation $y'' - 4y' + 4y = 2e^{2x}$. Find the general solution. Solve the initial value problem $y(0) = 0,\ y'(0) = 0$ by 2 methods (linear equation with constant coefficients, Newton' method). *(2+2)+2=6%*

5. Find the general solution of equation $y' = e^x (e^x - y)$. Solve the initial value problems $y(0) = 0$. Sketch its graph. *2+1=3%*

SOLUTIONS (without detailed explanations, figures).

1. Stable stationary points $y = \frac{\pi}{4} + k\frac{\pi}{2}$, $k = -1, +1$. Unstable stationary points $y = \frac{\pi}{4} + k\frac{\pi}{2}$, $k = -2,\ 0$. Inflection points $y = k\frac{\pi}{2}$, $k = \pm 2, \pm 1, 0$. *1+1,5=2,5%*

2. $y - 1 = \cdot -\frac{1}{c}(x - 2)$. Rays. $y' = \frac{y-1}{x-2}$. Orthogonal trajectories $(x-2)^2 + (y-1)^2 = c^2$. Circles.

3. Separable equation. Solution of the initial value problem $y(0) = 1$ is $y(x) = e^x$. For $y(0) = \frac{1}{2}$ the solution is $y(x) = \frac{1}{e^{-x} + 1}$. For $y(1) = 0$ the solution is $y(x) = 0$.

4. General solution $y = c_1 e^{2x} + c_2 x e^{2x} + x^2 e^{2x}$. The solution of the initial value problem $y = x^2 e^{2x} = x^2 + 2x^3 + 2x^4$. *(2+2)+2=6%*

5. General solution $y = ce^{-e^x} + e^x - 1$. The solution of the initial value problem $y = e^x - 1$. *2+1=3%*