Differential Equations 1, 2018. 12.11. 10:00. 90=15+75 minutes. K.F.51. 100%.

Good Luck!

<u>*Theory*</u> *3*(2+1)=9%.*

Name **3** scientists, who had important research related to Differential Equations as well and whose family names start with letter **G, E, P**. Write the main idea of the mathematical result (definition, theorem, etc.).

<u>*Exercises.*</u> *15+12+15+9=51%.*

1. Find the general solution for the equation $y' + 2x^3 = \frac{2}{x} y$, $0 < x$. Solve the initial value problem $y(2) = -8$. Check it. Study this function (range, roots, extrema, inflection points, graph, etc.). *10+5=15%.*

2. Consider the differential equation $y' = \frac{y^2 + y}{x}$, $0 < x$. Find the solutions of the following 3 initial value problems: $y(1) = -1, y(1) = -\frac{1}{2}$ and $y(1) = 0$. Sketch their graphs in 1 coordinate system. *8+4=12%*

3. Consider the second order equation $\ddot{x} + 2\dot{x} - 3x = 6$.

Solve the initial value problem $x(0) = -1, \dot{x}(0) = -3$ by 3 methods: (i) linear equation with constant coefficients, (ii) Laplace transformation, (iii) Newton's approximate method– until the 4th order terms.

Transform the equation to 2 dimensional system (by substitution $y = \dot{x}$). Find its solution by matrix method. Sketch the phase portrait. Find on the phase portrait the trajectory corresponding to the solution with initial conditions $x(0) = -1, y(0) = -3$. *5*3=15%*

4. Consider the Lyapunov stability of the trivial solution ($x = y = z = 0$) of system $\dot{x} = -2x + 2y$, $\dot{y} = -2y + 2z$, $\dot{z} = -y$ (by 2 methods (i) roots of the characteristic equation, (ii) Routh-Hurwitz criterion). *6+3=9%*

Vázlatos megoldások (ábrák nélkül).

1. Linear equation. General solution: $y = cx^2 - x^4$. Solution of the initial value problem $y(x) = 2x^2 - x^4$. Even function (but only part $0 < x$ is considered). Range $y \le 1$. $\lim_{x \to 0+} y(x) = 0$. Root $x_1 = \sqrt{2}$. Maximum $x_2 = 1$. Inflection point $x_3 = \frac{\sqrt{3}}{3}$. *15+10=25%*

2. Separable equation. The solution of the initial value problem $y(1) = -1$ is $y(x) = -1$. The solution of the initial value problem $y(1) = -\frac{1}{2}$ is $y(x) = -\frac{1}{x+1}$. The solution of the initial value problem $y(1) = 0$ is $y(x) = 0$. *8+4=12%*

3. General solution $x = c_1 e^t + c_2 e^{-3t} - 2$. The solution of the initial value problem $x = e^{-3t} - 2 = -1 - 3t + \frac{9}{2}t^2 - \frac{9}{2}t^3 + \frac{27}{4}t^4 + \dots$. The general solution of the system $\begin{pmatrix} x \\ y \end{pmatrix} = c_1 e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-3x} \begin{pmatrix} 1 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix}$. Saddle point. The trajectory is a ray. *5*3=15%*

4. Asymptotically stable. The characteristic equation (multiplied by -1) is $\lambda^3 + 4\lambda^2 + 6\lambda + 4 = (\lambda + 2)(\lambda^2 + 2\lambda + 2) = 0$. Roots $\lambda_1 = -2$, $\lambda_{2,3} = -1 \pm i$. *6+3=9%*