Differential Equations 1. Test 2. November 21, 2018. 6+5+3+3=17%. 90 minutes. Good Luck!

1. Solve the initial value problem $x(0)=3,\ \dot{x}(0)=1$ of the equation $\ddot{x}+2=x$ by 3 methods: (i) Laplace transformation, (ii-iii) equation with constant coefficients, find the articular solution with method of undetermined coefficients and variation of constants. *2+2+2=6%*

2. Consider the autonomous system of ODEs $\dot{x}=2y-2,\ \dot{y}=-2x+ay$. Determine the type of the equilibrium point (node, saddle, etc.), and sketch the phase portrait for the following values of the parameter a : $-5,\ 0$. Find the general solution as well. *2,5*2=5%*

3. Find the general solution of $\dddot{x}-\frac{3}{t}\ddot{x}+\frac{6}{t^2}\dot{x}-\frac{6}{t^3}x=0$, $t<0$.

4. Give the lowest order linear homogeneous differential equation with real constant coefficients, for which $x(t)=\cos^2(2t)+t$ is a solution. Find the general solution as well. Solve the initial value problem $x(0)=1,\ \dot{x}(0)=1$, $\ddot{x}(0)=0,\ \ldots, x^{(n-1)}(0)=0$, where n is the order of equation. *2+1=3%*

SOLUTIONS.

1. General solution $x(t)=c^1e^t+c^2e^{-t}+2$. The solution of the initial value problem is $x(t)=e^t+2$. *2+2+2=6%*
2. For $a=0$ the equilibrium point $(0,1)$ is a center ($\lambda_{1,2}=\pm 2i$), orientation is clockwise, the general solution is $x(t)=c^1\cos 2t+c^2\sin 2t$, $y(t)=1-c^1\sin 2t+c^2\cos 2t$. For $a=+5$ the equilibrium point $\left(-\frac{5}{2},1\right)$ is a stable node $\lambda_1=-1, \lambda_2=-4$. The general solution is $x(t)=-\frac{5}{2}-2c^1e^{-t}+c^2e^{-4t}$, $y(t)=1+c^1e^{-t}-2c^2e^{-4t}$. *2,5*2=5%*
3. $x(t)=c^1t+c^2t^2+c^3t^3$. *2,5+0,5=3%*
4. The equation is $x^{(IV)}+16\ddot{x}=0$ The general solution is $x(t)=c^1+c^2t+c^3\sin 4t+c^4\cos 4t$. The particular solution is $x(t)=1+t$. *2+1=3%*