

## **Mathematics A2 (BMETE90AX02).**

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### *Program*

Mathematics A2a -Vector Functions (of parts taught by Peter Moson).

Functions in several variables: continuity, differential and integral calculus, partial derivatives, Young's theorem. Local and global maxima/minima. Vector-vector functions, their derivatives, Jacobi matrix. Integrals: area and volume integral.

Infinite series: convergence, divergence, absolute convergence. Sequences and series of functions, convergence criteria, power series, Taylor series.

Fourier series: expansion, odd and even functions.

### *Literature:*

**Thomas' Calculus** by Thomas, G.B. et al. Addison-Wesley, 2005. (ISBN0321185587).

Hungarian translation is available: [http://www.interkonyv.hu/konyvek/Thomas\\_kalkulus\\_3](http://www.interkonyv.hu/konyvek/Thomas_kalkulus_3)

Test 1. Functions of several variables. March 14, 2018. 13%,

Test 2. Functions of several variables (continuation). Functional sequences, series. May 8, 2018. 13%.

## **Detailed Program.**

*2018-02-06.*

Introduction.

Functions of 2 real variables. Domain of definition, range, graph. Partial derivatives.

Examples  $f(x,y)=1-x-y$ ,  $(1-x^2-y^2)^{1/2}$ ,  $x^2+y^2$ ,  $x^2-y^2$ . Tangent plane.

*Homework:* Thomas. Chapter 14, §1, Exercises 7, 9. Chapter 14, §3, Exercises 7, 9, 43.

Chapter 14, §6, Exercises 9, 11.

*2018-02-07.*

Functions of 2 real variables. Taylor polynomial. Tangent plane. Local extremum (necessary, sufficient condition).

Examples for local extremum  $f(x,y)=x^2+y^2$ ,  $x^2-y^2$ ,  $x^3-3xy+y^3$ .

*Homework:* Thomas. Chapter 14, §7, Exercise 25.

*2018-02-13.*

Functions of 2 real variables. Global extremum (on compact sets).

Examples:  $f(x,y)=x^2+y^2-2y$  on  $x^2+y^2\leq 4$  and on  $-1\leq x\leq 2$ ,  $0\leq y\leq -x+2$ .

Investigation on the boundary by its parametrization.

*Homework:* Thomas. (Chapter, paragraph, exercise). 14.7.31, 14.7.41.

*2018-02-14.*

Functions of 2 real variables. Level curves. Examples:  $f(x,y)=1-x-y$ ,  $(1-x^2-y^2)^{1/2}$ ,  $x^2+y^2$ ,  $x^2-y^2$ ,  $c=-1, 0, +1$ . Gradient. Chain rule (gradient is perpendicular to the level curve). Directional derivative (max, min – gradient).

*Homework:* Thomas. (Chapter, paragraph, exercise). 14.5.9, 14.5.17.

2018-02-21.

Functions of 2 real variables. Conditional extremum. Examples: (i)  $\max/\min f(x,y) = x^2 + y^2 - 2y$  on  $x^2 + y^2 = 4$ . (ii) The maximum, minimum distance between  $x^2 + y^2 = 1$  and the point  $P(3,4)$ .

Homework: Thomas. (Chapter, paragraph, exercise). 14.8. 1, 5.

2018-02-27.

Double integrals on normal domains.

Normal domains with respect to  $x, y$ . Order of integration.

Examples. (i) Volume of a pyramid calculated by 2 methods (elementary, integration).  $f(x,y) = 1 - x - y$ ,  $D: 0 < x < 1, 0 < y < 1 - x$ . (ii) Integration of  $f(x,y) = xy$  on  $0 \leq x \leq 1, x^2 \leq y \leq x$  (by 2 methods: directly, changing the order of integration).

Substitution. Jacobi determinant. General case, polar coordinates.

2018-02-28.

Separable double integrals.

Substitution (polar coordinates). Volume of the sphere (by elementary method, by the integration of  $f(x,y) = (1 - x^2 - y^2)^{1/2}$ ,  $D: x^2 + y^2 < 1$ ).

Improper integrals. Example -  $f(x,y) = \exp(-x^2 - y^2)$ ,  $D: 0 < x, y < \infty$ .

Application of double integrals (volume, mass, center of gravity, moment of inertia of plates). Example: Integral of  $f(x,y) = x^2 + y^2$  over the unit circle. Different meanings (volume, mass, moment of inertia).

Homework. Thomas. 15.1. 1, 5, 21, 31. 15. 3. 1, 3, 7.

2018-03.06.

Functions of 3 real variables. Level surfaces. Gradient. Tangent plane to the level surface.

Directional derivative. Local extremum (necessary condition). Examples:

$f(x, y, z) = x + y + z, x^2 + y^2 + z^2, x^2 + y^2 - z^2$ . Conditional extremum. Closest distance examples (point to plane, cylinder, and sphere).

2018-03.07.

Practical lecture. Solution of Test 1 (2017-03-14).

2018-03.13.

Practical lecture. Solution of Retake Test 1 (2017-05-10). Consultation.

2018-03.14. Wednesday, 16:15. K. building 255.

Test 1.

2018-03-20.

Triple integration in Cartesian coordinates. Volume of a pyramid. Triple integration in cylindrical coordinates. Volume of the cone. Triple integration in spherical coordinates. Volume of the sphere.

2018-03.21.

Remarks on Test 1. Finalization of results.

Applications of triple integrals in Cartesian, spherical, cylindrical coordinates. Mass, center of gravity, moment of inertia. Example: Moment of inertia of a (spinning top like) homogeneous body (calculated in cylindrical, spherical coordinates).