BME Mathematics Global Exam June 13, 2017. 11:00. ChMax. 40%. 15-75 minutes.

Good Luck!

*<u>Theory</u> (3*5=15%.). Max. 15 minutes.*

1. Define the $\arcsin$ (inverse sine) function. Sketch the graph, give the derivative and the second order Maclaurin polynomial.

2. Polar coordinates (definition, Jacobi determinant).

3. Continuous functions. Formulate at least 2 theorems related to $f \in C^0_{[a,b]}$.

<u>Exercises</u> 35+25+25=85%. Min. 75 minutes.

1. Solve the initial value problem $y(0)=0,\ y'(0)=2$ for the differential equation $y''+y'=2e^x+1$ by 3 methods: (i) linear equation with constant coefficients (ii) reducible second order equation, (iii) Newton's method (write the 4th order Maclaurin polynomial of the solution). *7+3*6=25%*
Sketch the graph of this solution. *10%*

2. Find the farthest point of the set $D=\{(x,y,z) \mid x^2+y^2+z^2=9\}$ to the point $P=\left(2\sqrt{2},2\sqrt{3},4\right)$ by 2 methods (i) elementary geometry, (ii) conditional extremum. How much is the distance? *10+10+5=25%*

3. Consider the vector field $\vec{v}(\vec{r})=\vec{j}\times\vec{r}$, $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$.
(i) Calculate $\oint_\gamma \vec{v}(\vec{r})d\vec{r}$, if $\gamma: x=2\cos t,\ y=0,\ z=\sin t,\quad 0\le t\le 2\pi$ by 2 methods (definition, Stokes theorem). What kind of curve is γ? *10+10=20%*
(ii) Is $\vec{v}(\vec{r})$ a potential vector field? Calculate $\oiint_F \vec{v}(\vec{r})d\vec{F}$, if the closed surface is the boundary of the unit sphere (normal points outward):*2+3=5%*

Solutions

1.General solution $y=c_1+c_2e^{-x}+e^x+x$. The solution of the initial value problem is $y=e^x+x-1=2x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\dots$. *5*5=25%.*

$x=0$ is a root. There is no extremum, inflection point. Asymptote $y=x-1$, $x\to-\infty$. *10%.*

2. The farthest point $Q=\left(-\sqrt{2},-\sqrt{3},-2\right)$. *10+10=20%.* $PQ=9$. *5%*

3. (i) $\vec{v}(\vec{r})=\vec{j}\times\vec{r}=z\vec{i}-x\vec{k}$. $rot\vec{v}(\vec{r})=2\vec{j}$. $\int_\gamma \vec{v}(\vec{r})d\vec{r}=-4\pi$. The curve is an ellipse. *10+10=20%*

(ii) There is no potential function. $rot\vec{v}(\vec{r})=2\vec{j}$. $div\vec{v}(\vec{r})=0$. $\oiint_F \vec{v}(\vec{r})d\vec{F}=0$ according to the Gauss-Ostrogradsky theorem.. *2+3=5%*