Mathematics EP1 (BMETE90AX33)

Lecturer: *Dr. Peter Moson* (www.math.bme.hu, +3614632690, +36309329626)

Detailed program (2017):

Lecture 1. (September 6, Wednesday)
Real numbers. Axioms.
Functions of 1 real variable (domain, range, graph, even, odd, etc.).
Elementary functions of 1 real variable (polynomials, exponential, logarithm, sine, cosine, sine hyperbolic, and cosine hyperbolic).

Lecture 2. (September 8, Friday)
Elementary functions of 1 real variable (inverse functions – trigonometric, hyperbolic).
Definition of limit, derivative of functions of 1 real variable.
Rules of derivation. Table of derivatives.
Homework. Thomas. Chapter 3. §2. Ex. 1, 5, 9. §4. Ex. 1, 3, 9.

Lecture 3. (September 13, Wednesday)
Applications of derivation.
Linearization – tangent lines (example: $x^2 + y^2 = 2$, $(x_0, y_0) = (1,1)$).
Necessary condition of maximum, minimum. Extrema for polynomial functions (examples: (i) maximal area of a rectangular garden - $f(x) = x(2-x)$, (ii) maximal volume of a box without roof - $f(x) = x(4-2x)^2$).
Homework: Thomas 3.8.3,7,11. , 4.4.9,17.

Lecture 4. (September 15, Friday)
Necessary, sufficient conditions for extrema. Convex, concave functions, inflection points. Sketching the graphs of functions (examples: $f(x)=2x^3-3x^2$, $f(x) = \sin x + \sin^2 x$).
Homework: Thomas 4.4. 11, 23,35.

Lecture 5. (September 22, Friday)
L'Hospital's rule ("0/0" case with proof). Limits of functions on the boundary of the domain of definition. Asymptotes (horizontal, vertical). Asymptotes (oblique). Sketching the graphs of functions (examples: f(x)=xlnx, $f(x) = \dfrac{x^3}{3x^2+1}$).
Homework: Thomas 4.4.35. 4.6.11,23.

Lecture 6. (September 27, Wednesday)
Sketching the graphs of functions $f(x) = \dfrac{e^x}{x}$, $f(x) = \arctan x^2$, $f(x) = \sin x / x$.

Lecture 7. (September 29, Friday)
Power series.

Geometric series ($\frac{1}{1-x}=\sum_{k=0}^{\infty}x^k$, $|x|<1$), convergence. Polynomials, identities. Development of basic elementary functions into Maclaurin series: $e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}$, $\sin x=\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+1}}{(2k+1)!}$, $\sinh x=\sum_{k=0}^{\infty}\frac{x^{2k+1}}{(2k+1)!}$, $\cos x=\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k}}{(2k)!}$, $\cosh x=\sum_{k=0}^{\infty}\frac{x^{2k}}{(2k)!}$.

Lecture 8. (October 6, Friday)
Calculation of values (e.g. $e=\sum_{k=0}^{\infty}\frac{1}{k!}$), limits (e.g. $\lim_{x\to 0}\frac{\sin x}{x}=1$) of functions by power series.
Homework: Thomas 11.9.1, 7, 13.

Lecture 9. (October 11, Wednesday).
Indefinite integral. Methods of integration (elementary – table of integrals, by parts, by special substitutions).

Lecture 10. (October 13, Friday)
Solution of Sample Test1.

Lecture 11. (October 25, Wednesday)
Test 1. Solution of Test 1.

Lecture 12. (October 27, Friday)
Definite integral. Definition. Calculation of integrals by the Newton-Leibniz formula.
Applications of integration (area, arc length, center of gravity of homogeneous plates, volume – surface area of bodies of revolution). Example (for all problems: triangle, cone, triangular plate - $f(x)=-x+3$, $0\le x\le 3$.
Homework: Thomas 6.1.45/a and 6.4.13 , 15.

Lecture 13. (November 8, Wednesday)
Improper integrals. Examples: $\int_1^{+\infty}\frac{1}{x^2+1}dx=\frac{\pi}{2}$. $f(x)=\frac{1}{x}$, $1\le x$ - area (divergent), volume of the rotated body around the x-axes.
Ordinary differential equations of the 1st order. Theorem of existence and uniqueness. Examples: $y'=y, y(0)=1$, $y'=1+y^2, y(0)=0$, . $y'=3y^{\frac{2}{3}}, y(0)=0$. Separable ordinary differential equations of the 1st order. Solution of the previous examples.

Lecture 14. (Friday, November 10)
Separable and linear ordinary differential equations of the 1st order. General solution, solutions of initial value problems, sketching their graphs. (Lecture is based on Thomas' Calculus Chapter 9, Paragraphs 1, 2, 5.
Homework. Thomas 9.1.11, 9.2.1, 9.5.17.

Lecture 15. (Wednesday, November 15)
Vectors in spatial geometry.
Addition, scalar (dot), vector (cross) products of vectors, their meaning (projection, area) and calculation in Cartesian coordinates.

Definition of determinants. Calculation of area, volume by determinants.

Lecture 16. (November 17, Friday).
Solution of Sample Test 2.

Lecture 17. (November 22, Wednesday).
Test 2.

Lecture 19. (November 29, Wednesday).
Discussion of Test 2.
Summary of planar, spatial geometry. Vectors, Cartesian coordinates. Dot (scalar), cross (vector) product.
Matrices. Determinants, addition, multiplication of matrices.

Lecture 20. (December 1, Friday).
Remake. Test 1, Test 2. Consultation for the remaining students (exam).

Literature:

Thomas' Calculus by Thomas, G.B. et al. Several editions, e.g. Addison-Wesley, (ISBN0321185587)