DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS **NAME:**
BMETE90MX46 **NEPTUN Code:**

February 8, 2012, 10:45-11:35, Test (for information – non-graded)

Please use another pieces of paper for the detailed explanation of the solution.

Part 1. Please solve the following equations.

1. Find the solution $y = y(x)$ of the initial value problem $y(0) = 2$, $y'(0) = 1$ for $y'' + y' - 2y + 2 = 0$. Sketch its graph.

2. Solve the initial value problem $x^2 y' + \cos^2 y = 0$, $y(1) = \frac{\pi}{4}$. Sketch the graph of this solution.

Part 2. Please mark (underline, encircle) your answer.

3. Have you ever solved systems of ordinary differential equations? **YES** **NO**

   If your answer was YES, then find the general solution of the linear system $\begin{aligned} \dot{x} &= x + 2y \\ \dot{y} &= 3x + 2y \end{aligned}$.
   Sketch the phase portrait. Solve the initial value problem $x(0) = 1$, $y(0) = -1$. Sketch the corresponding trajectory.

4. Have you ever solved partial differential equations? **YES** **NO**

   If your answer was YES, then solve the following boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial^2 x}$, $\frac{\partial u(t,0)}{\partial x} = \frac{\partial u(t,\pi)}{\partial x} = 0$, $u(0,x) = \frac{1}{2} - \sin^2 x$.

5. Have you ever heard about the Laplace transformation? **YES** **NO**

   If your answer was YES, then solve the initial value problem $y(0) = 2$, $y'(0) = 1$ for $y'' + y' - 2y + 2 = 0$ by using Laplace transformation.

6. Have you ever heard about the Banach fixed-point theorem? **YES** **NO**

   If your answer was YES, then please formulate it.

Good Luck !