Mathematics A3 for Mechanical Engineers – Homework 1

Deadline: 4th week (send the solved homework in electronic form to mate.frks@gmail.com or hand it in on paper on the lectures)
For consultation appointments about the corrected homeworks write an e-mail to mate.frks@gmail.com.

In every exercise, sketch the solution (or several members of the family of the solution).

1. Solve the initial value problem

$$y'x = y + x^2$$
$$y(1) = 2.$$

On which interval(s) is the solution defined?

2. Solve the differential equation $y'' = \frac{y'}{x} + x\sin x$

3. Solve the differential equation $\sqrt{x}\frac{dy}{dx} = \mathrm{e}^{y+\sqrt{x}}$.

4. Find the ortogonal trajectories of the family of the curves $y = cx^2$.

5. The so called Bernoulli differential equation is an ordinary differential equation of the form $y' = p(x)y + q(x)y^\alpha$. If $\alpha \neq 1$ and $\alpha \neq 0$, the equation is not linear. Although, with the substitution $u = y^{1-\alpha}$ one can transform the equation into a linear one, as:

$$u' = (1-\alpha)y^{-\alpha}y' \quad \rightarrow \quad \frac{y'}{y^\alpha} = \frac{u'}{1-\alpha} = p(x)y^{1-\alpha} + q(x).$$

Thus $u' = (1-\alpha)p(x)u + (1-\alpha)q(x)$, which is linear in the dependent variable u. Using this technique, solve the following differential equation:

$$y' - x^3y^3 = xy$$