Mathematics A3 for Mechanical Engineers – Homework 2

Send the solved homework in electronic form to mate.frks@gmail.com or hand it in on paper on the lectures
For consultation appointments about the corrected homeworks write an e-mail to mate.frks@gmail.com.

1. Derive the wave equation ($\Delta\mathbf{E} = \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial^2 t}$, where $c = \sqrt{\mu_0\epsilon_0}$) from the source-free Maxwell's equations!

$$div\mathbf{E} = 0$$
$$rot\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$
$$div\mathbf{B} = 0$$
$$rot\mathbf{B} = \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

2. Calculate the divergence and rotation of the following vector field:

$$\mathbf{v}(\mathbf{r}) = (x^2 - y^2)\mathbf{i} + (y^2 - z^2)\mathbf{j} + (z^2 + x^2)\mathbf{k}$$

3. Central force field Evaluate the divergence and rotation of a central force field: $\mathbf{v}(\mathbf{r}) = f(r)\mathbf{r}$ where $r = \sqrt{x^2 + y^2 + z^2}$, and $f(r)$ is an arbitrary function ($f(r) : \mathbb{R}^+ \to \mathbb{R}$).

4. Evaluate the line integral of $\mathbf{v}(\mathbf{r})$ on the curve γ:

$$\mathbf{v}(\mathbf{r}) = 2x\mathbf{i} - \frac{y}{2}\mathbf{j} + (x - z)\mathbf{k}$$
$$\gamma(t) = (1 - t)\mathbf{i} + (t^2 - 1)\mathbf{j} + t^2\mathbf{k} \qquad 0 \le t \le 2$$

5. Evaluate the line integral of $\mathbf{v}(\mathbf{r}) = y\mathbf{i} - x\mathbf{j}$ on the parabola $y = \sqrt{(1 - x)}$

6. Evaluate the surface integral of $\mathbf{v}_1(\mathbf{r}) = r^2\mathbf{r}$ and $\mathbf{v}_2(\mathbf{r}) = \frac{\mathbf{r}}{r^3}$ on a sphere centered in the origo with radius 1.

7. Consider the following vector field:

$$\mathbf{v}(\mathbf{r}) = \frac{ay}{(x-y)^2}\mathbf{i} + \left(\frac{2x}{(x-y)^2} + 1\right)\mathbf{j} + z\mathbf{k}$$

Calculate a if the vector field has a potential function. Evaluate the potential function! Using the potential theory, evaluate the line integral of the vector field on a semi-circle between $(-1, 0, 0)$ and $(1, 0, 0)$ in the plane xy.

8. Evaluate the surface integral of $\mathbf{v}(\mathbf{r}) = (x-y)\mathbf{i} + (x+y)\mathbf{j} + xy\mathbf{k}$ on the surface $z = xy$ where $x^2 + y^2 \leq 4$. Show that Stokes' theorem isn't always helpful...